ROBUST DESIGN OF MULTILEVEL SYSTEMS USING DESIGN TEMPLATES

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ROBUST DESIGN OF MULTILEVEL SYSTEMS USING DESIGN TEMPLATES

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The LORD *is my strength and shield. I*
trust him with all my heart. He helps me,
and my heart is filled with joy. I burst
out in songs of thanksgiving.
Psalm 28:7

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LIST OF SYMBOLS AND ABBREVIATIONS

$A_{cross\text{-}section}$	Cross-sectional area
$A_i(x)$	System achievement in cDSP
a	Characteristic cross-section dimension
B	In-plane spacing of webs of square honeycomb core
BRP	Blast resistant panel
cDSP	Compromise decision support problem
cp_i	Control point at location i
d_i^-, d_i^+	Deviation variables in cDSP
δ, δ_{max}, $\Delta\delta$	Deflection, maximum deflection, variation of deflection
E	Young's Modulus
ε_c	Average crushing strain of BRP core
F	Force
g	Gravitational constant
G_i	System goal in cDSP
g_{δ}, Δg_{δ}	BRP deflection constraint function, variance in deflection constraint
g_M, Δg_M	BRP mass constraint function, variance in mass constraint
g_{SH1}, Δg_{SH1}	BRP front face shear constraint 1, variance in front face shear constraint 1
g_{SH2}, Δg_{SH2}	BRP front face shear constraint 2, variance in front face shear constraint 2
Γ_{SH}	BRP front face shear constraint 1 value
$H, \overline{H}$	Thickness of undeformed and deformed BRP core layer, respectively
h_c, h_f, h_b	Thickness of core webs and face sheets, respectively
HD-EMI$_i$	Hyper-Dimensional Error Margin Index in i performance direction
h_{total}	Total height of BRP
I_0	Impulse for BRP loading

KE_I, KE_{II}	BRP kinetic energy per unit area in panel after stages 1 and 2, respectively
L	Length of cantilever beam, length of BRP
λ_c, λ_s	Factors governing strength of BRP core in crush and stretch
m	Mass
M, ΔM	BRP mass / area, variation in BRP mass / area
μ_p, Δ_p	Mean and standard deviation of impulse peak pressure
μ_t, Δ_t	Mean and standard deviation of impulse load characteristic pulse time
p_0	Peak pressure of free-field pulse
ρ	Density
$\rho_{f,}\rho_{b,}\rho_c$	Density of front face sheet, core, and back face sheet of a BRP
$\Delta\rho$	Variation in density
R_c	Relative density of BRP core layer
s.f.	Safety factor
σ_y	Yield strength
$\sigma_{y,f,}\sigma_{y,b,}\sigma_{y,c}$	Yield strength of front face sheet, core, and back face sheet of a BRP
$\Delta\sigma_y$	Variation in yield strength
T_i	Target performance
t_0	Characteristic time of incident pressure pulse
vf_i	Volume fraction at location *i*
w	Weight ($w = mg$)
w_i	Weight of design goal *i*
W_p^{III}	Plastic work per unit area dissipated in stage three
Z	Deviation function in cDSP

GLOSSARY OF KEY TERMS

The Glossary of Key Terms presents fundamental definitions in the context of multilevel design, robust design, and template-based design with contributions from Matthias Messer and Stephanie Thompson of the Systems Realization Laboratory at Georgia Tech.

Blast Resistant Panel (BRP) – (*noun*) a sandwich structure consisting of solid front and back face sheets surrounding a honeycomb core. Under impulse loading, BRPs are designed to experience less deflection than solid plates of equal mass.

Deductive Design Solution – (*noun*) in a multilevel design problem, a solution that is obtained by transferring design information from the most specific model to the most general model. Also called a bottom-up design approach.

Design – (*verb*) to systematically plan out a product or process, often in graphic form. To create or contrive a product or process for a particular purpose or effect (compiled from www.dictionary.com).

Design Complexity – (*noun*) a measure of the interactions and couplings in a system model denoted by the number of independent design variables (degrees of freedom) used to describe system performance.

Design Freedom – (*noun*) the extent to which a system can be modified while still meeting design requirements (Simpson, et al. 1996)

Design Template – (*noun*) a pattern, used as a guide in making decisions in a design process. A design process having a preset format, used as a starting point for a particular design application so that the format does not have to be recreated each time it is used (Panchal, et al. 2004; compiled from www.dictionary.com)

Inductive Design Solution – (*noun*) in a multilevel design problem, a solution that is obtained by transferring design information from the most general model to the most specific model. Also called a top-down design approach.

Level – (*noun*) a position or plane in a graded scale of values. An extent, measure, or degree of intensity. A division of a multilevel design problem representing the precision of system performance models used in making design decisions (compiled from www.dictionary.com).

Materials Design – (*noun*) the process of tailoring material properties to meet design goals. Also referred to multiscale materials design, denoting the various length scales at which material performance is modeled and design decisions are made (e.g., nanometer scale, micrometer scale)

Model Abstraction – (*noun*) the extent to which system behavior is modeled using generalizations to approximate actual, concrete system phenomena (compiled from www.dictionary.com).

Model Fidelity – (*noun*) the adherence of system performance models to actual system behavior. As model abstraction *increases*, model fidelity *decreases*. In general, as design complexity *increases*, model fidelity *increases*.

Multilevel Design – (*noun*) a subset of engineering design methods in which design problems are defined and analyzed at various levels of design complexity.

Multiscale Design – (*noun*) a subset of engineering design methods in which design problems are defined and analyzed at various length and / or time intervals.

Robust Design – (*noun*) a method for improving the quality of products and processes by reducing their sensitivity to variations, thereby, reducing the effect of variability without removing its sources (Seepersad 2004; Taguchi 1986; Taguchi and Clausing 1990).

Scale – (*noun*) a length or time interval at which system performance models are designed in order to predict system behavior in a multiscale design process.

Satisficing – (*adj*) a term describing a solution that may be sub-optimal, but sufficiently meets system requirements (Simon 1996).

SUMMARY

PROBLEM: Traditional methods in engineering design involve producing solutions at a single level. However, in complex engineering design problems, such as concurrent product and materials design, extensive design space exploration at a single level is cumbersome if not impossible. Therefore, to encourage design space exploration, complex engineering design problems can be divided and analyzed at various levels of model complexity, known as a multilevel design approach. One example of multilevel design is the design of a material, product, assembly, and system. However, it is observed that analyzing design problems at multiple levels increases the possibility for introducing and propagating uncertainty. Therefore, critical needs of multilevel design processes include the organization and simplification of complex design information and the management of propagated design uncertainty.

APPROACH: Design templates are reusable, modular design process units that can be applied to a variety of design problems. One way to address the critical needs of multilevel design is the application of a template-based design approach with multilevel design methods. Additionally, it is advantageous to infuse multilevel robust design techniques in a template-based multilevel design environment. Design solutions that perform predictably in the presence of uncertainty are robust designs. The Inductive Design Exploration Method (IDEM) is an existing design method used to produce robust multilevel design solutions. In this thesis, a multilevel design template with robust design goals is created based on the Compromise Decision Support Problem and IDEM. The verification and validation of the multilevel design template is examined using the Validation Square construct which involves investigation of the theoretical and performance capabilities of the multilevel design template.

RESEARCH QUESTIONS:

Primary Research Question – How can information regarding multilevel robust design processes be captured and stored in a reusable format?

Primary Research Hypothesis – Information regarding robust multilevel design processes can be captured and stored in a reusable format by developing generic, reusable, computer executable design templates based on the Compromise Decision Support Problem (cDSP) and Inductive Design Exploration Method (IDEM).

Secondary Research Question – How can information regarding the robust multilevel design of blast resistant panels by captured and stored in a reusable, computer-executable format?

Secondary Research Hypothesis – By particularizing a generic multilevel design template for the multilevel robust design of blast resistant panels and translating design process information to computer-interpretable modules, information regarding the robust multilevel design of blast resistant panels can be captured and stored in a reusable, computer-executable format.

CONTRIBUTIONS: In this thesis, the possibilities of a template-based approach to multilevel design are explored. A multilevel design template for producing inductive multilevel robust solutions for complex engineering design problems is developed and validated. The multilevel design template is particularized and applied to two example problems including the multilevel design of a cantilever beam and its associated material and the multilevel design of a blast resistant panel. Based on the successful application of the multilevel design template to example problems, confidence is built in the ability to apply the template to additional multilevel engineering design problems.

CHAPTER 1

FOUNDATIONS OF MULTILEVEL DESIGN

The motivation for this thesis is to investigate the robust design of multilevel systems using design templates. As stated in the glossary of key terms, multilevel design is a subset of engineering design in which design problems are defined and analyzed at various levels of complexity. Additionally, template-based design is a strategy for simplifying a design process by using predefined design templates to support design decision-making. In this thesis, a design template is developed for a multilevel design environment, and applied to two multilevel design problems. In order to design systems that are insensitive to variation, robust design concepts are embedded in the multilevel design template.

Much of the current research in multilevel design relates to multiscale materials design in which material properties are tailored in order to meet specific design requirements. In a multiscale materials design process, material behavior is predicted based on design and analysis models at various length and time scales (e.g., length [continuum, mesometer, micrometer, nanometer, etc.]; time [second, microsecond, nanosecond, etc.]). The term "multiscale" is used to denote the different length and time scales of material performance analysis in a materials design process. However, in this thesis, the term "multilevel" rather than "multiscale" is adopted in order to convey a broader design concept. Instead of limiting investigation to length and time measurements, in this thesis, measures of design complexity in a multilevel design process are analyzed. Therefore, the terms "multilevel" and "design level" are used to convey the idea of a design process in which performance analysis tools are developed to predict system behavior at various levels of model complexity.

At the beginning of each chapter, a figure and table are presented in order to give a summary of the information in the current chapter (Table 1.1), and to show how the current chapter relates to remaining thesis chapters (Figure 1.1).

Table 1.1 – Summary of Chapter 1

Heading / Sub-Heading	**Information**
Multilevel Design – A Framework for Solving Complex Design Problems	
Multilevel Design of Engineering Systems	Multilevel design overview: - A design process divided into levels of model complexity - Example multilevel design problem—aircraft design
Multilevel Design Challenges	Challenges in multilevel design: - Partitioning a complex design problem - Characterizing level-to-level interactions - Deductive vs. inductive solution paths - Multilevel robust design approach
Frame of Reference	
Robust Design of Multilevel Systems	Summary of robust design concepts applied to multilevel design problems
Template-Based Design Approach	Summary of template-based design approach in engineering design
Template-Based Design of BRPs	BRP design: - Overview of motivating example for this thesis - Template-based design approach applied to BRP design - BRP collaboration team
Research Focus and Contributions	
Research Questions and Hypotheses	- Primary research question and hypothesis - Secondary research question and hypothesis
Research Contributions	Research contributions in this thesis: - Development of a template-based approach to multilevel design - Multilevel robust design solution to BRP design problem
Method Validation Strategy – The Validation Square	
Verifying and Validating Design Methods	Motivation for method validation
Thesis Validation Strategy	Validation Square: - Description of Validation Square construct - How to apply Validation Square to a thesis - Validation strategy for this thesis
Overview of Example Problems	
Design of a Cantilever Beam and its material	Overview of cantilever beam example problem
Design of a BRP	Overview of BRP example problem
Chapter 1 Synopsis	

Chapter 1 begins with a description of multilevel design and its challenges. In Section 1.2, the frame of reference for this thesis is established with a discussion of robust design of multilevel systems and a template-based design approach. In Section 1.3, the research focus of this thesis is presented, including the primary and secondary research questions and hypotheses. Research contributions from this thesis are also discussed. The design method validation strategy used in this thesis is presented in Section 1.4. Chapter 1 concludes with an overview of the example problems completed in this thesis.

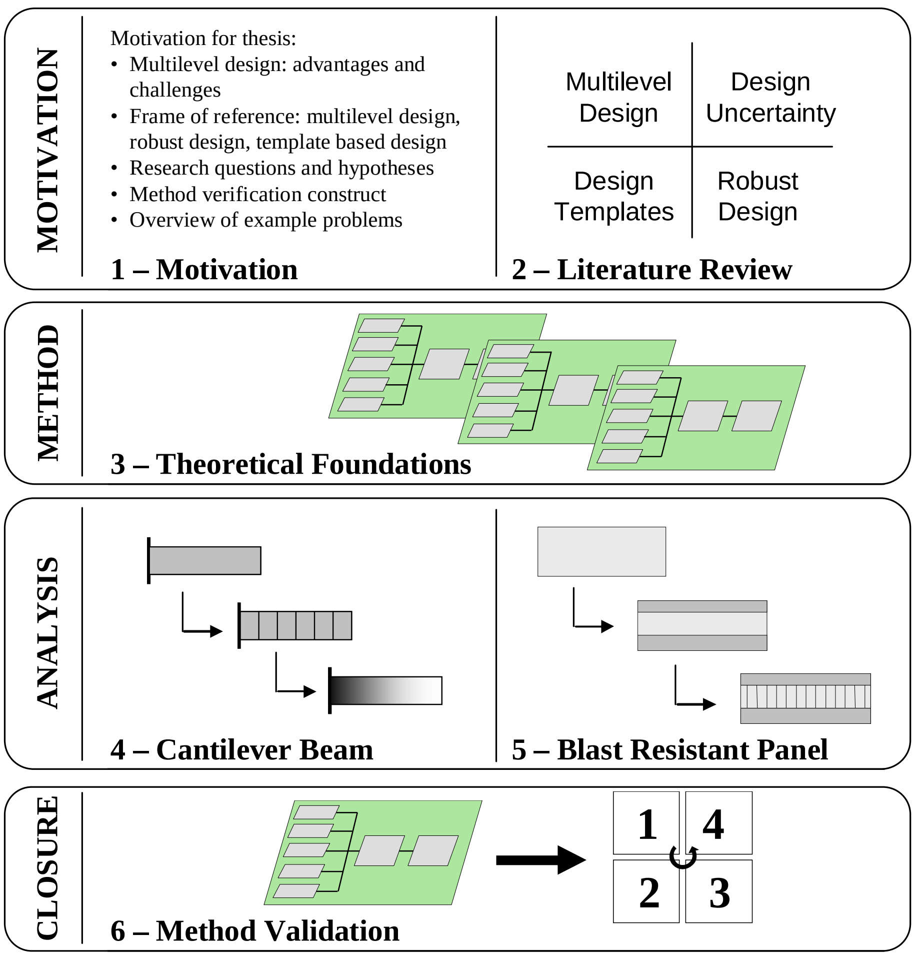


Figure 1.1 – Setting the context for Chapter 1

1.1 MULTILEVEL DESIGN – A FRAMEWORK FOR SOLVING COMPLEX DESIGN PROBLEMS

Multilevel design is an approach for managing design complexity. With the current trend of increasing product performance requirements, many design problems are too complex to allow for thorough and agile design space exploration given the current computational tools. By applying a multilevel design approach, prohibitively complex design problems are divided into levels of manageable complexity. Design prediction models are created at each level, and design information is passed among all levels. Overall design decisions are made by combining design information at each design level. In Section 1.1 an overview of multilevel design and the key challenges faced in multilevel design are given.

1.1.1 Multilevel Design of Engineering Systems

As stated in the glossary of key terms, multilevel design is a subset of engineering design methods in which design problems are defined and analyzed at various levels of design complexity. Multilevel design processes begin with a complex design problem with many design variables. In order to fully explore the extensive design space defined by many design variables, the complex design problem is divided according to levels of model complexity. The complexity of design levels is directly related to the number of design variables used to describe system performance. Design information among various levels is combined in making overall design decisions.

For complex design problems, a multilevel design approach is preferred over traditional design methods in order to mange vast design complexity and to limit the use of unnecessarily complex prediction models in decision-making. The system prediction models of a multilevel design process vary in complexity and, therefore, computation cost. In a multilevel design approach, complex models are only referred to when such prediction accuracy is needed. Otherwise, less complex (and less computationally expensive) prediction models are used in design design-making.

An example of multilevel design is shown in the aircraft design problem in Figure 1.2. At the least complex design level, overall system specifications (aircraft dimensions, total mass, geometry, etc.) are used to model aircraft performance (velocity, drag, thrust, etc.). As model complexity increases, aircraft subsystems are considered when modeling overall aircraft performance. At this more complex design level, the relationship between individual aircraft subsystems and overall aircraft performance is modeled. Increasing in design model complexity, each component and part in the aircraft system is modeled in order to predict overall aircraft performance. As prediction models continue to increase, material specifications of each part are modeled in determining overall system performance. The most complex design level considers the quantum characteristics of each material in predicting overall aircraft performance.

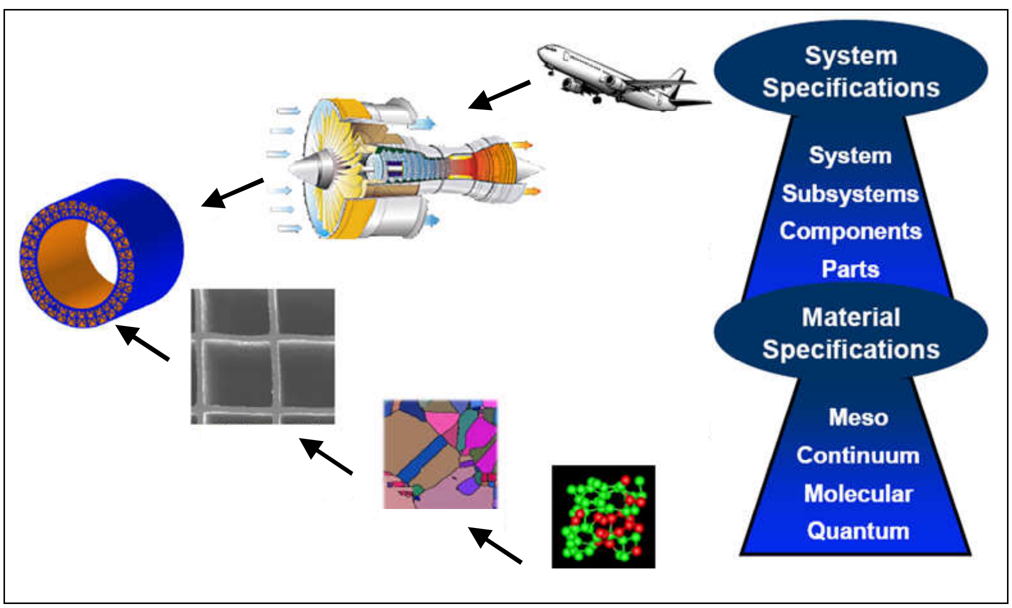


Figure 1.2 – Multiscale design process at product and material levels (Seepersad 2004)

Aircraft design presents significant design complexity with thousands, if not millions, of design variables. With current computational limitations, it is not feasible to holistically model, analyze, and design an aircraft while simultaneously considering all design variables. By implementing a multilevel design approach, aircraft performance models are developed at various levels of design complexity and design decisions are made at each level. Design information is then shared with other design levels and overall system design decisions are determined.

1.1.2 Multilevel Design Challenges

Several of the key challenges in multilevel design are presented in Section 1.1.2. These multilevel design challenges are addressed in a multilevel design template presented in Chapter 3. Multilevel design challenges are as follows:

Partitioning a complex design problem – When presented with a complex design problem, it can be difficult to know how to divide the design problem according to levels of model complexity. The main challenge is in determining how much detail in system performance models is needed in order to reach a valuable overall design solution. With most product design problems, it is unnecessary to model material specifications below the continuum level. However, as concurrent product and materials design increases in popularity, detailed material models are needed in system prediction models.

Level-to-level interactions – An additional multilevel design challenge exists in sharing design information among various design levels. Mapping functions must be developed to describe the relationship between design variables at each level. Additionally, in a computational design environment, the appropriate computational infrastructure must be developed in order to pass design data among various design levels.

Deductive vs. inductive solution paths – Once system performance has been sufficiently modeled at each level, design information at each level is used to make design decisions regarding the overall system. In order to reach an overall design solution, the designer must choose between two solution paths: deductive or inductive. As defined in the glossary of key terms, a deductive design solution is achieved by making design decisions progressing from specific design models to general design models. In contrast, an inductive design solution is achieved from analyzing design information from general to specific design models. In engineering design, inductive solution paths are preferred because this approach allows design goals to be stated at the beginning of a design process and details regarding the achievement of these goals to be presented at the end of a design process.

Multilevel robust design – The distributed and collaborative nature of multilevel design causes it to be susceptible to propagated uncertainty in the design and analysis process chain. When design information at various levels is generated and analyzed using different design tools in different locations, potential error is introduced as design information is transferred. In order to achieve a robust multilevel design solution, uncertainty must be modeled and managed. In Chapter 3, a method for multilevel robust design is implemented in a template-based approach to multilevel design. Robust design concepts are discussed in more detail in Section 1.2.1, Chapter 2 and Chapter 3.

1.2 FRAME OF REFERENCE

In Section 1.2, the frame of reference for this thesis is presented with a discussion of multilevel robust design and template-based design. More details regarding each of these topics are presented in a literature review in Chapter 2. Additionally, the motivating example for this thesis, the design of a blast resistant panel, is presented.

1.2.1 Robust Design of Multilevel Systems

Robust design is the practice of improving the quality of products by reducing sensitivity to noise factors, including uncertainty. When products are robust, performance levels remain stable despite the presence of noise factors (Taguchi 1986; Taguchi, et al. 1990). A robust solution may have lower performance levels than an optimum solution in the absence of variation; however, a robust solution produces predictably satisfactory results in the presence of variation. In multilevel design problems where the likelihood of uncertainty introduction and propagation is high, robust solutions are often favored.

The concept of designing for robustness was made popular by Taguchi (Taguchi 1986). Taguchi recognized that some noise factors could not be controlled; therefore, designs should be robust to these uncontrollable variations. Rather than increase the cost of a product by trying to eliminate noise factors, Taguchi proposed to minimize the variance of performance as well as bringing the mean on target. Uncertainty in multilevel design problems arises from noise factors, uncertain control factors, uncertain system models, and propagated process chain uncertainty. The concepts of robust design proposed by Taguchi have been adapted to a multilevel robust design method, the Inductive Design Exploration Method (IDEM) (Choi, et al. 2005). In IDEM robust solutions are selected by minimizing response variation while maximizing distance to design variable bounds. IDEM is the base method for a template-based approach to multilevel robust design developed in Chapter 3. A formal review of robust design is presented in Chapter 2.

1.2.2 Template-Based Design Approach

Design templates are reusable design process modules that support decision-making at various stages in design. In order to facilitate reuse, design templates are created with sufficient generality so that they can be applied to a variety of design problems. Design templates supporting decisions at various stages in design can be linked to accommodate the specific needs of individual design processes. Design templates can be created at various ranges of abstraction. At a high level of abstraction, design templates resemble generic decision support structures that can be applied at any stage of the design process, in any design domain. Design templates at a lower level of abstraction can take the form of computer executable modules developed for a specific type of design problems (Panchal, et al. 2004).

The principle goal in this thesis is to develop a design template to support multilevel robust design. In this thesis, design templates at various levels of abstraction are created, starting with a generic design template, and then progressing to a design template for solving a specific design problem. In Chapter 3, a design template at a high level of abstraction is developed to demonstrate the concepts involved in a multilevel robust design process. When solving the example problems in Chapter 4 and Chapter 5, the general multilevel design template is particularized for application in the specific example problems.

1.2.3 A Template-Based Approach to the Robust Design of Blast Resistant Panels

The motivating example in this thesis is the design of a blast resistant panel. Blast resistant panels (BRPs) are sandwich structures consisting of solid front and back face sheets surrounding a honeycomb core. An example of the type of BRPs designed in this thesis is shown in Figure 1.3.

As shown in Figure 1.3, the front face sheet receives the initial pressure loading from a blast. The topology of the core is designed to dissipate a majority of the impulse energy in crushing. The back face sheet provides additional protection from the blast as well as a means to confine the core collapse and absorb energy in stretching.

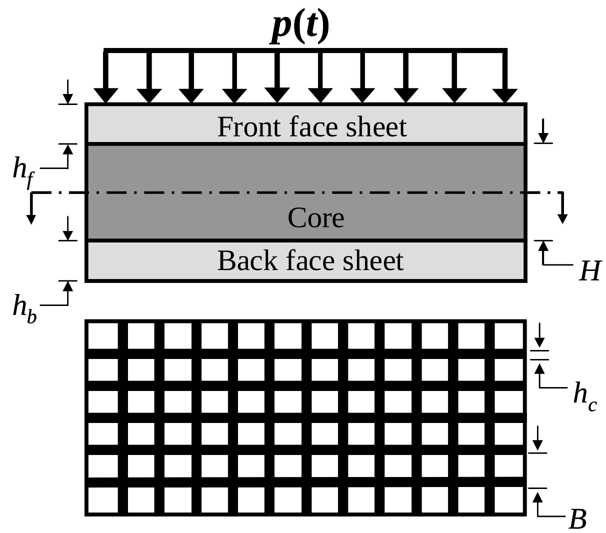


Figure 1.3 – Schematic of BRP sandwich structure under uniform impulse loading

Due to the complex nature of a BRP, it is advantageous to implement a multiscale robust design approach in solving this design problem. Additionally, a reusable template-based BRP design process is beneficial in exploring various potential alterations in BRP design including the addition of a fill material in core cells, expanding the number of panel layers, and exploring new material systems. The multilevel BRP design problem provides the motivation for developing a template-based approach to multilevel robust design, presented in Chapter 3. The successful implementation of the developed multilevel design template in BRP design adds value to the validation of the developed template, a topic that is discussed in more detail in Section 1.4. The BRP design problem is a collaborative effort among students of the Systems Realization Lab at Georgia Tech. In Figure 1.4, students involved in BRP design, along with individual research interests, are presented.

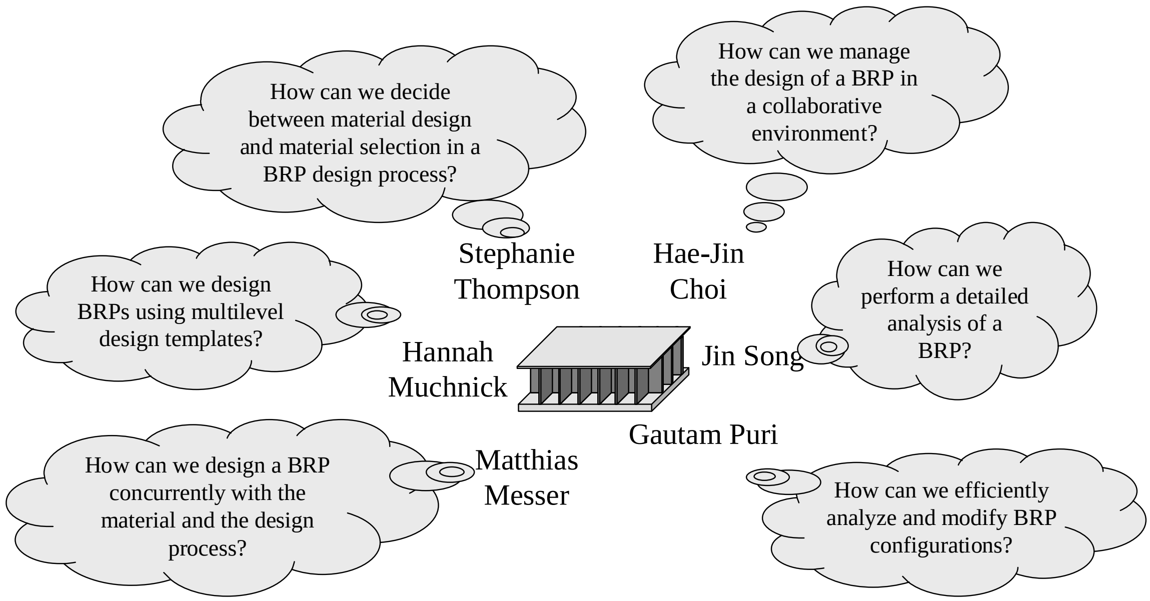


Figure 1.4 – BRP design team with individual research goals

1.3 RESEARCH FOCUS AND CONTRIBUTIONS

The primary and secondary research questions and hypotheses addressed in this thesis are given in Section 1.3. The research questions are formulated out of the need to develop a multilevel design template for application in BRP design.

1.3.1 Research Questions and Hypotheses

The primary research question relates to the development of a multilevel design template to provide a reusable, adaptable framework for completing the design of multilevel systems. In the primary research hypothesis, it is proposed that the multilevel design template be based on an existing multiscale robust design method, IDEM (Choi, et al. 2005). The primary research question and hypothesis are listed below.

Primary Research Question

How can information regarding multilevel robust design processes be captured and stored in a reusable format?

Primary Research Hypothesis

Information regarding robust multilevel design processes can be captured and stored in a reusable format by developing generic, reusable, computer executable design templates based on the Compromise Decision Support Problem (cDSP) and Inductive Design Exploration Method (IDEM).

The secondary research question relates to the implementation of the developed multilevel design template in the design of blast resistant panels. The secondary research question and hypothesis are listed below.

Secondary Research Question

How can information regarding the robust multilevel design of blast resistant panels by captured and stored in a reusable, computer-executable format?

Secondary Research Hypothesis

By particularizing a generic multilevel design template for the multilevel robust design of blast resistant panels and translating design process information to computer-interpretable modules, information regarding the robust multilevel design of blast resistant panels can be captured and stored in a reusable, computer-executable format.

The primary research question is addressed with the development of a multilevel design template (Chapter 3); the secondary research question is addressed in the implementation of the robust design template in the multilevel design of BRPs (Chapter 5). The verification and validation of the multilevel design template is evaluated using the Validation Square Construct (see Section 1.4). A visual representation of how the research questions are addressed in the remainder of this thesis is given in Figure 1.5.

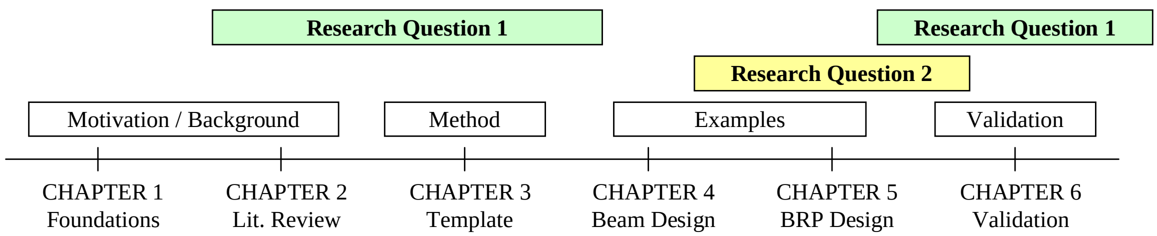


Figure 1.5 – Addressing the research questions throughout thesis

1.3.2 Research Contributions

The main contributions presented in this thesis are the development of a multilevel design template and its application in the robust multilevel design of a BRP. The research contributions are realized in addressing the primary and secondary research questions. Details regarding research contributions in this thesis are presented in Section 1.3.2 and analyzed in Section 6.1.

Multilevel Robust Design Template

In Chapter 3, a multilevel design template is presented. The multilevel design template is developed from a template-based approach to engineering design and an existing multilevel robust design method, IDEM. The design template is a reusable pattern to support decision-making in multilevel design problems. The multilevel design template is a contribution to the field of engineering design because it combines two design approaches, multilevel robust design and template-based design, in order to create a design tool to capture and store multilevel robust design information in a reusable format. As it is presented in Chapter 3, the multilevel design template is sufficiently general such that it can be applied to a variety of multilevel design problems. In Chapter 4 and Chapter 5, the developed design template is particularized into computer-executable modules for solving two example problems, the design of a cantilever beam and its material and the design of a BRP.

Robust, Multilevel BRP Design

The motivating example in this thesis is the robust design of a BRP. The second main research contribution in this thesis is the particularization and application of the multilevel design template for the robust design of a BRP such that BRP design information is captured and stored in a reusable, computer-interpretable format. The design approach presented in the generic multilevel design template (Chapter 3) is adapted to computer-executable modules to aid in design decisions in the multilevel robust design of a BRP (Chapter 5).

1.4 METHOD VALIDATION STRATEGY – THE VALIDATION SQUARE

In this thesis, the verification and validation of the multilevel design template is assessed using the Validation Square construct. The Validation Square is a tool used to ease the leap of faith required to move from theory to practice in engineering design methodology. The progression of building confidence in the usefulness of the method based on the Validation Square is broken into four stages and is shown in Figure 1.6 (Seepersad, et al. 2005). A review of method validation using the Validation Square is presented in Section 1.4. In Section 1.4.1, an overview of method validation is presented, and in Section 1.4.2, a strategy for validation and verification of this thesis is presented.

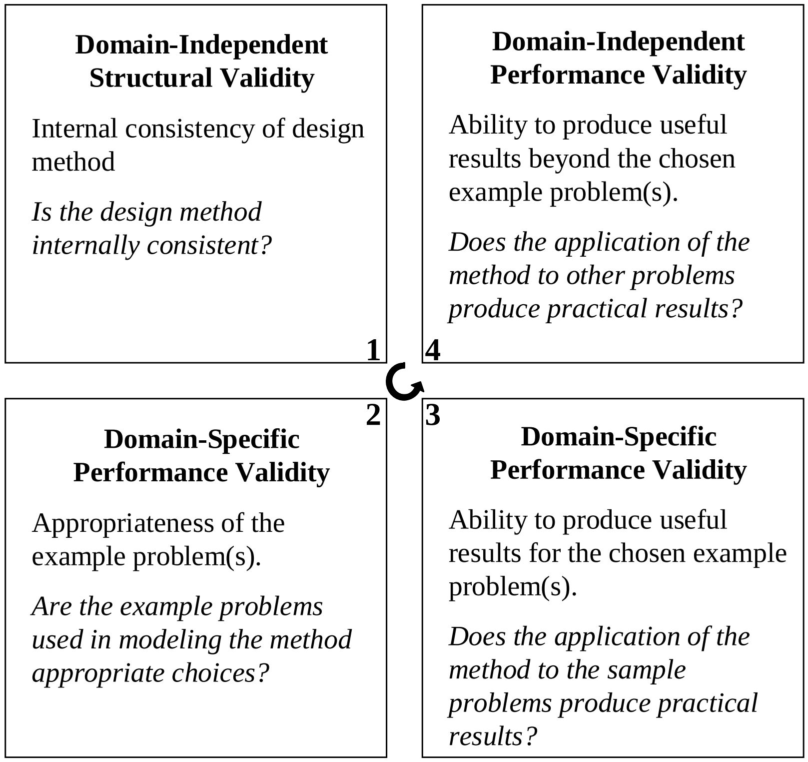


Figure 1.6 – Validation Square construct

1.4.1 Verifying and Validating Design Methods

Section 1.4.1 on method validation using and the Validation Square is leveraged with minor modification from the Ph.D. dissertation of Carolyn Conner Seepersad (Seepersad 2005).

Validation—justification of knowledge claims, in a modeling context—of engineering research has typically been anchored in formal, rigorous, quantitative validation based on logical induction and/or deduction. As long as engineering design is based primarily on mathematical modeling, this approach works well. Engineering design methods, however, rely on subjective statements as well as mathematical modeling; thus, validation solely by means of logical induction or deduction is problematic. Pedersen and coauthors and Seepersad and coauthors propose an alternative approach to validation of engineering design based on a relativistic notion of epistemology in which “knowledge validation becomes a process of building confidence in its usefulness with respect to a purpose."

The Validation Square is a framework for validating design methods in which the 'usefulness' of a design method is associated with whether the method provides design solutions correctly (structure validity) and whether it provides correct design solutions (performance validity). Additionally, the validity of the method itself (domain-independent) and the method applied to example problems (domain-specific) is addressed. This process of validation is represented graphically in Figure 1.7.

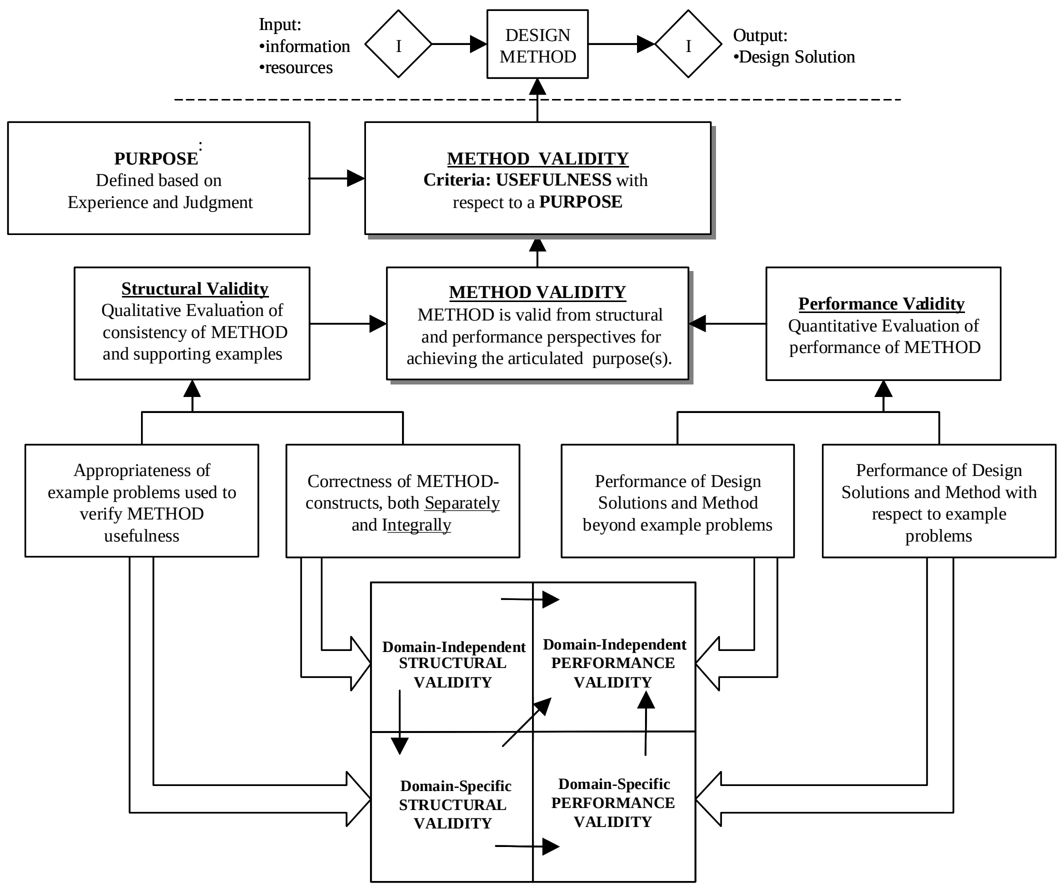


Figure 1.7 – Design method validation: a process of building confidence in usefulness with respect to a purpose (Seepersad, et al. 2005)

With respect to the quadrants of the Validation Square, domain-independent structure validity involves accepting the individual constructs constituting a method as well as the internal consistency of the assembly of constructs to form an overall method. Domain-specific structure validity includes building confidence in the appropriateness of the example problems chosen for illustrating and verifying the performance of the design method. Domain-specific performance validity includes building confidence in the usefulness of a method using example problems. Domain-independent performance validity involves building confidence in the generality of the method and accepting that the method is useful beyond the example problems.

How can this validation framework be implemented in a thesis?

Establishing *domain-independent structure validity* involves searching and referencing the literature related to each of the parent constructs utilized in the design method. In addition, flow charts are often useful for checking the internal consistency of the design method by verifying that there is adequate input for each step and that adequate output is provided for the next step. A list of criteria may be useful for establishing and comparing the domain-independent structure validity of methods and constructs with respect to a set of explicit, favorable properties.

Establishing *domain-specific structural validity* consists of documenting that the example problems are similar to the problems for which the methods/constructs are generally accepted, that the example problems represent actual problems for which the method is intended, and that the data associated with the example problems can be used to support a conclusion.

Domain-specific performance validity can be established by using representative example problems to evaluate the outcome of the design method in terms of its usefulness. Metrics for usefulness should be related to the degree to which the method's purpose has been achieved (e.g., reduced cost, reduced time, improved quality). It is also important to establish that the resulting usefulness is, in fact, a result of applying the method. For example, solutions obtained with and without the construct/method can be compared and/or the contribution of each element of the method can be evaluated in turn. An important part of domain-specific performance validity is empirical verification of data used to support domain-specific performance validation. Empirical verification can be established by demonstrating the accuracy and internal consistency of the data. For example, in optimization exercises, multiple starting points, active constraints and goals, and convergence can be documented to verify that the solution is stationary and robust. For any engineering model it is important to verify that data obtained from the model represents aspects of the real world that are relevant to the hypotheses in question. The model should react to inputs in an expected manner or in the same way that an actual system would react.

Domain-independent performance validity can be established by showing that the method/construct is useful beyond the example problem(s). This may involve showing that the problems are representative of a general class of problems and that the method is useful for these problems; from this, the general usefulness of the method can be inferred.

1.4.2 Thesis Validation Strategy

In Table and 1.2 and Figure 1.8, an outline of the validation strategy for this thesis is presented. It is arranged according to the quadrants in the Validation Square, and references are included for chapters in which method validation is documented. In Chapter 3 – Chapter 5, contributions to method validation are presented at the end of each chapter. A detailed summary of the validation of the multilevel design template is given in Chapter 6.

Table 1.2 – Validation strategy implemented in this thesis

Domain-Independent Structural Validity The multiscale robust design template is based on existing multiscale robust design method (IDEM). The internal consistency of the template is largely dependent on the internal consistency of the base method. The internal consistency of the base method and multiscale design template are considered in (Chapter 2, Chapter 3).
Domain-Specific Structural Validity The appropriateness of selected problems is based on the ability of the examples to test the various aspects of the multiscale design template. That is, the example should be clearly defined and multiscale in nature. The appropriateness of example problems is address (Chapter 4, Chapter 5).
Domain-Specific Performance Validity The reasonableness of design results when applying the multiscale template is investigated by observing the value of design solutions obtained from completing the example problems. Additionally, the value that the design template adds to a multiscale design process is discussed in (Chapter 4, Chapter 5).
Domain-Independent Performance Validity The likelihood of producing desirable results when applying the design template to other multiscale design problems is investigated by considering the generic, mutable nature of the template and determining the type of design problems for which the design template is best suited. Discussed in (Chapter 3, Chapter 6)

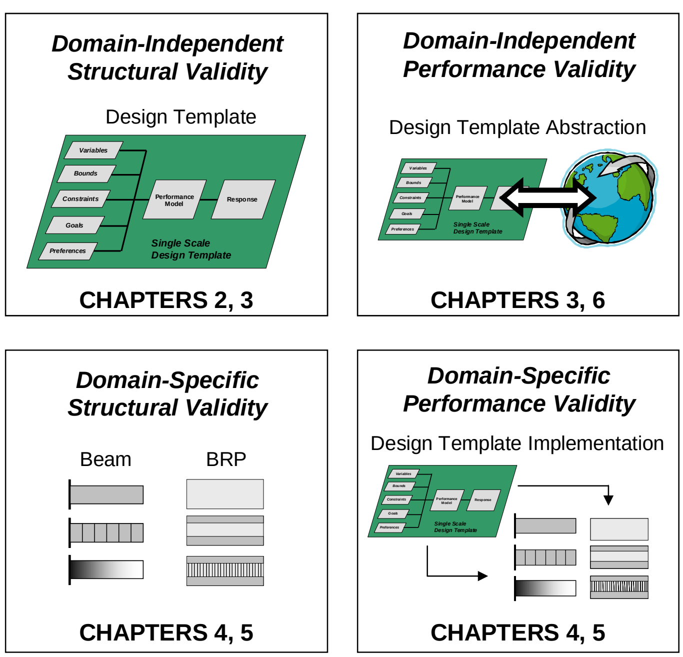


Figure 1.8 – Validation strategy implemented in this thesis

1.5 OVERVIEW OF EXAMPLE PROBLEMS

In this thesis, two example problems relating to multilevel robust design using design templates are presented and solved. The example problems are chosen based on their appropriateness in relation to the primary and secondary research questions. The example problems play a valuable role in the verification and validation of a design template approach for the robust design of multilevel systems. Successfully applying design templates to the example problems *and* observing useful results builds confidence in the validity of the developed multilevel design template. An overview of the example problems is given in Section 1.5 and illustrated in Figure 1.9.

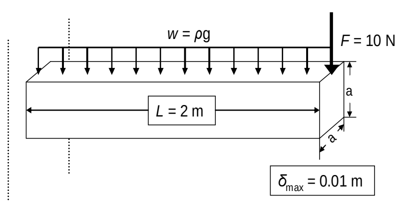
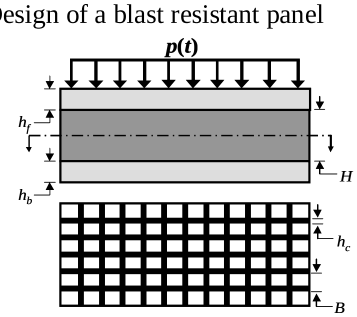
Example Problem	Motivation
Cantilever Beam Design Design of a cantilever beam and its associated material 	• Design goals: minimize beam weight; maximize beam robustness to uncertainty in material properties • Design constraints: maximum deflection $\delta_{max} \leq 1$ cm; safety factor ≥ 1 • Design variables: material properties of beam; beam cross-sectional area • Example problem used to illustrate the concepts of template-based multilevel robust design and to validate developed multilevel design template
Blast Resistant Panel Design Design of a blast resistant panel 	• Design goals: minimize BRP deflection & mass; maximize BRP robustness to uncertainty in material properties and loading conditions • Design constraints: maximum deflection $\delta_{max} \leq 0.1$L; mass / area ≤ 100 kg/m^2; additional constraints • Design variables: material properties of BRP; dimensions of BRP • Example problem intended as a comprehensive illustration of template-based multilevel robust design and to validate developed multilevel design template

Figure 1.9 – Overview of example problems

1.5.1 Overview of Conceptual Example – Design of a Cantilever Beam

The conceptual example problem involves the design of a cantilever beam and its associated material, presented in Chapter 4. The loading conditions, variable bounds, and performance constraints are given in the design problem. The design goals include minimizing beam mass while maximizing beam robustness to uncertainty in material properties. Since the beam example contains the concurrent design of product and material, the example problem is easily identified as a multilevel design problem. The multilevel design template is particularized for use in solving the cantilever beam design problem. The design process and results obtained add value to the verification and validation of the developed template based approach to the robust design of multilevel systems. The conceptual example problem is selected because it is useful in illustrating the concepts of multilevel robust design using design templates. This conceptual example problem is relatively simple, and is used to demonstrate the ideas discussed in the thesis without unneeded difficulties of solving a complex example problem.

1.5.2 Overview of Comprehensive Example – Design of a Blast Resistant Panel

A second example, more comprehensive in nature, is presented and solved in Chapter 5. The comprehensive example is used to illustrate the effectiveness of the developed template-based approach to multilevel design problems for typical problems encountered in a non-idealized engineering design process. The comprehensive example problem involves the design of a BRP. The loading conditions, variable bounds, and performance constraints are given in the design problem. The design goals are to minimize BRP deflection, minimize BRP mass / area and maximize system robustness to uncertainty in loading conditions and materials properties. The various levels of geometric complexity of the design problem cause the design problem to be multilevel in nature. The multilevel design template developed in Chapter 3 is particularized and applied to the BRP design problem. As with the conceptual example, the successful completion of the comprehensive example problem builds confidence in the validity of the developed template based approach to multilevel design problems.

1.6 SYNOPSIS OF CHAPTER 1

At the conclusion of each chapter, a chapter synopsis is given. The chapter synopsis is intended to summarize the information presented in the previous chapter, and introduce topics discussed in the following chapter. Chapter 1 provides the introduction and motivation for this thesis. Chapter 1 begins with a discussion of multilevel design and its challenges. Then, the frame of reference for this thesis—robust design and template-based design—is discussed. Next, the research focus is presented including the research questions and hypotheses and research contributions. The strategy for method validation in this thesis is presented with a discussion of the Validation Square. Chapter 1 concludes with an overview of the two example problems that are completed in this thesis in order to add value to the verification and validation of a template-based approach to multilevel robust design.

Chapter 2 continues the establishment of the motivation and frame of reference of this thesis with a review of relevant topics in design literature in order to identify areas of research opportunity. The identified research gaps in Chapter 2 directly correlate with the research questions presented in Chapter 1. In Chapter 3, the theoretical foundations of this thesis are developed with the creation and discussion of a multilevel design template. In Chapters 4 and 5 the multilevel design template is applied to two example problems—the design of a cantilever beam and it associated material and the design of a BRP. In Chapter 6 aspects of method validation presented throughout the thesis are brought together with a thorough assessment of the validity of the multilevel design template. Chapter 6 also contains the research contributions of this thesis and opportunities for future research.

CHAPTER 2

MULTILEVEL TEMPLATE-BASED ROBUST DESIGN – REVIEW OF LITERATURE AND IDENTIFICATION OF RESEARCH GAPS

In Chapter 2, key concepts of template-based multilevel robust design are presented. To begin, a discussion of multilevel design and materials design as a multilevel design process is presented. Then, topics relating to uncertainty and robust design are discussed. Uncertainty classification and previously developed robust design methods are presented. Then, information regarding a template-based approach to design is presented. With each reviewed topic, an explanation of its value added to the work presented in this thesis is given. At the end of Chapter 2, areas for development in template-based multilevel robust are identified. The research gap is the motivation for formulating the primary and secondary research questions.

The work presented in this thesis is intended to illustrate the extension and implementation of existing engineering design concepts, rather than the development of new fields of study for template-based multilevel robust design. Therefore, topics such as multilevel design, uncertainty and robust design, template-based design, and method validation are given in the reviewed literature in order to present the foundation from which the work in this thesis begins. Significant portions of Chapter 2 are adapted from the work of former and current graduate students in the Systems Realization Laboratory, with specific contributions identified in the text. In Section 2.4, research opportunities in multilevel robust design are identified based on the current state of research discussed in Section 2.1 – Section 2.3. A summary of Chapter 2 is given in Table 2.1. An illustration of the topics presented in Chapter 2 in the context of the entire thesis is shown in Figure 2.1.

Table 2.1 – Summary of Chapter 2

Heading / Sub-Heading	**Information**
Multilevel Design	
Definition of Multilevel Design	Multilevel design: - Definition - Aircraft design example
Materials Design – A Multilevel Design Problem	Materials design overview including: - Definition - Methods to achieve materials design - Materials design process - Key requirements in materials design - Requirements list for multilevel robust design process
Uncertainty and Robust Design	
Uncertainty in a Design Process	Design uncertainty: - Non-parametric system noise - Un-configured system noise
Robust Design Definition	Robust design overview
Robust Design Classification	Robust design classification – (Type I, Type II, Type III, multiscale robust design)
Design Templates	
Design Templates in Engineering Design	Definition of design template as related to engineering design
Requirements for Design Templates	Key requirements: - Reusable - Modular - Mutable - Archival
The Compromise Decision Support Problem – A Design Template	Presentation of cDSP and robust design techniques using the cDSP
Research Opportunities	
Research Opportunities Relating to Multilevel Robust Design – From Multiscale Design to Multilevel Design	Extending the usefulness of the concepts in multiscale materials design to include all complex engineering design problems
Research Opportunities Relating to a Template-Based Approach to Multiscale Robust Design	Adapting robust design methods (specifically, IDEM) to a template-based design environment
Chapter 2 Synopsis	

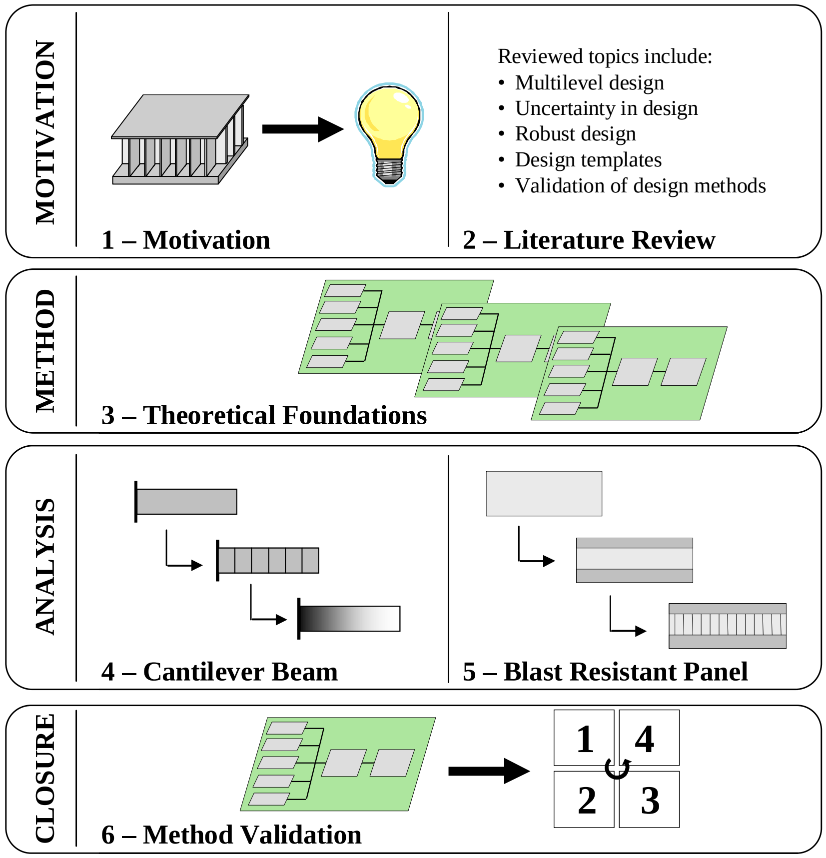


Figure 2.1 – Setting the context for Chapter 2

2.1 MULTILEVEL DESIGN

In Section 2.1 a review of multilevel design literature is given in an effort to identify future research opportunities in this field. Since the term "multilevel design" is most commonly known as "multiscale design" in engineering design literature, Section 2.1.2 contains a discussion of multiscale materials design, a subset of multilevel design. In Section 2.4 areas for future development in multilevel design are discussed.

2.1.1 Definition of Multilevel Design

Multilevel design is gaining particular interest in the engineering design community due to increased complexity and more demanding performance requirements in engineering design problems. Recall from Chapter 1 that multilevel design is a subset of engineering design in which design problems are defined and analyzed at various levels of complexity. Due to the relatively new nature of multilevel design concepts, a common taxonomy for the study of multilevel design has not been widely adopted in the engineering design community. Therefore, basic definitions for multilevel design are provided in the glossary of key terms. Contributions to the glossary of key terms were provided by Matthias Messer and Stephanie C. Thompson of the Systems Realization Lab.

Recall from Section 1.1.1 (Figure 1.2) in which the design of an aircraft and its associated material is given as an example of a multilevel design problem. Figure 1.2 is reproduced in Figure 2.2, with additional details added.

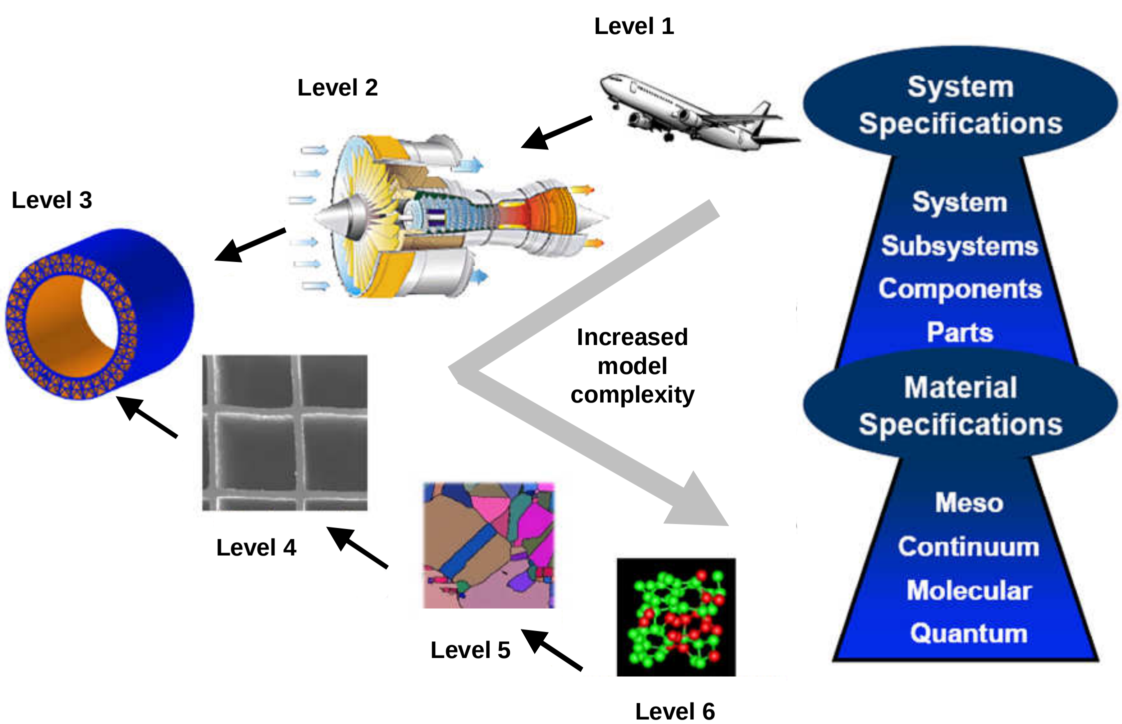


Figure 2.2 – The design of an aircraft and it associated material – a multilevel design problem. Modified from: (Seepersad 2004)

In Figure 2.2, the illustrated multilevel design process contains six levels of model complexity. At Level 1, the aircraft is modeled as a single product with overall product specifications (dimensions, mass, propulsion, etc.). As one moves to Level 2, aircraft subsystems are considered in system performance models. Modeling aircraft performance becomes much more complex when systems and subsystems are considered due to an increasing amount of detail considered in product performance models. As design complexity increases, material specifications are considered at Level 4 to Level 6, representing the materials design portion of this design problem. By increasing the complexity with which system performance is predicted (i.e., increasing design level) system performance is more accurately predicted, although the cost of performance models increases. The concept of modeling and designing a multilevel design problem using levels of model complexity is a key topic addressed in this thesis.

2.1.2 Materials Design – A Multilevel Design Problem

As shown in Figure 2.2, materials design is part of a multilevel design process (often called multi*scale* materials design in the materials design community). Materials design is the process of tailoring material properties to meet the requirements of specific design problems (Seepersad 2004). Materials can be tailored or adapted to produce new materials with specific properties and performance levels. Examples of materials design are given in Figure 2.3 and include topology design and functionally grading material properties. In the past, new materials were created largely by a process of experimental trial and error, and new materials were often discovered by chance. Today researchers are in the process of defining a systematic design method for integrated product and materials design. Materials design is investigated in this thesis because multiscale material modeling and design, a well-developed research field, contains many similarities to the idea of multilevel modeling and design discussed in this thesis.

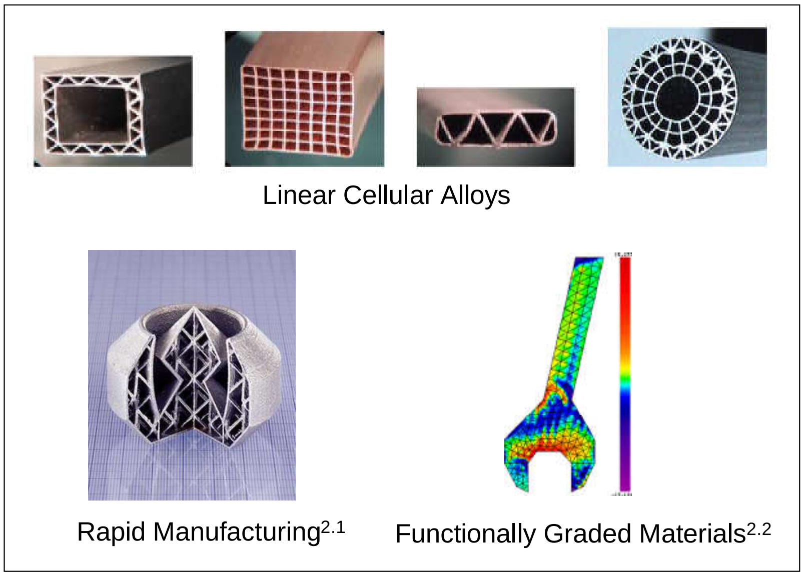


Figure 2.3 – Current materials design research. Modified from: (Seepersad 2004)

Designing new materials is complicated by the fact that the performance of a material is determined by several inter-related characteristics (processing, structure, property, performance) as shown in Figure 2.4. The processing link represents the manufacturing processes used to create a material. The structure link represents the microstructure of the material. The processing path directly affects a material's microstructure. A material's microstructure is identified by (for example) grain size and distribution. The property link in the chain in Figure 2.4 represents the physical properties of the material. The microstructure of a material directly impacts the properties of the material. Material properties describe the behavior of a material and can be found in many engineering material tables (for example, Young's Modulus, density, and thermal conductivity). The performance of a material describes how a part constructed from the given material behaves under specific conditions.

[2.1] Image from: Laser Center Flanders, *http://www.lcv.be/nl/nieuwe_ontw.asp?id=123&oper=show rubriek_id=10* [cited February 3, 2007]
[2.2] Image from: University of Oslo, *http://www.math.uio.no/avdb/faststoff/felt_eng.shtml* [cited February 3, 2007]

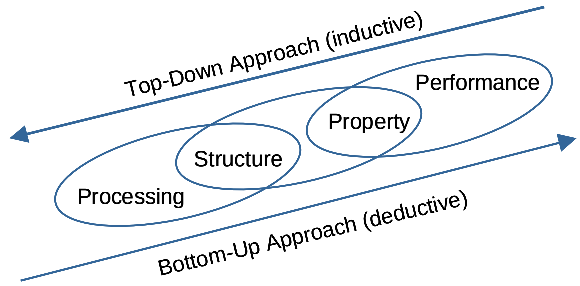


Figure 2.4 – Materials design process. Modified from: (Olsen 1997)

Current materials design processes are deductive in nature (bottom-up). Changing the processing path of a material adjusts its microstructure. Adjusting the microstructure of a material changes the properties and performance of the material. From an engineering design perspective, it is advantageous for materials design processes to consist of an inductive (top-down) approach in which designers specify the required material performance at the beginning of the design process. Then the material properties, microstructure, and processing path will be determined based on the material performance requirements.

A definition for materials design in the context of this thesis is given in the glossary of key terms. Critical needs in a materials design problem, unique from design problems that take place at a single level, are listed below (based on research collaboration with Matthias Messer and Stephanie C. Thompson of the Systems Realization Lab). In a materials design process, due to the varying levels of design complexity used in predicting material performance, it is shown that materials design is a multilevel design process. The critical needs of a materials design process is abstracted in the development of a requirements list for a multilevel design process, given in Table 2.2.

Critical needs in a multiscale materials design process:

- Quasi unlimited design freedom in material configuration and composition

- Strong couplings between subsystems, multiple disciplines, and physical phenomena at various scales
- Coupling between differing physical phenomena at various scales must be explicitly modeled and accounted for in decision making
- Fundamentally different types of models appear at various scales and model refinement may lead to fundamentally different types of models
- Significantly greater complexity than in conventional systems implying the necessity of designer expertise
- Uncertainty generation and propagation within and throughout scales must be understood, modeled, and managed
- The need for a single design process that holistically describes the overall design variables, bounds, constraints, and goals in a multiscale materials design process
- Complexity displayed in a unmanageably large number of design variables which inhibits extensive and agile design space exploration

Table 2.2 – Requirements list for a multilevel design process based on critical needs in a multiscale materials design process

Requirements list for a multilevel design process			**Issued On: 1/23/2007**
Problem Statement: Design a multilevel design process that facilitates design decision-making at various levels of model complexity, and addresses the critical needs identified in a multiscale materials design process (see text in previous paragraph for critical needs)			
#	**Demand/Wish**	**Requirements**	
Overall Design Process			
1	D	Encompasses design information and decision-making at various levels of model complexity	
2	D	Multilevel design process is sufficiently general to ensure successful application to a variety of multilevel design problems	
3	D	Provides the information infrastructure in order to pass design information among inherently different design models at various levels	
4	W	Increased complexity implies a greater need for designer expertise. Should be flexible to allow modifications from designer at any stage in design process	

Table 2.2 (continued) – Requirements list for a multilevel design process based on critical needs in a multiscale materials design process

Interactions and Couplings Among Levels		
5	D	Couplings among levels should be sufficiently understood and modeled
6	D	Interfaces between inherently different models should allow for design information flow
Managing Design Freedom		
7	W	Number of design variables should be limited to allow agile design space exploration at most complex design levels
Uncertainty Modeling and Management		
8	W	Uncertainty generation and propagation within and throughout scales should be understood, modeled, and managed

The concept of materials design as a multilevel design process is included in the reviewed literature because materials design is a key topic currently addressed in multiscale design research, and multiscale materials design is a specific class of multilevel design. In Section 2.4, opportunities for future research related to multilevel design are given. Additionally, the topic of product and materials design is addressed in the example problems in Chapter 4 and Chapter 5 in which concurrent product and material multilevel design problems are solved.

2.1.3 Multilevel-Multiscale vs. Multilevel-Homogenization Design Processes

Multilevel design is described in two ways: multilevel-multiscale design and multilevel-homogenization design. An example of multilevel-multiscale design relating to BRP design is shown in Figure 2.5 (a multilevel-multiscale design process is also shown in Figure 1.2 and Figure 2.2). Multilevel-multiscale design is a multilevel design process in which an increase in model complexity relates to a decrease in system length and / or time scale. As shown in Figure 2.5, BRP design is only one aspect of an overall multilevel design problem describing tank design. In Figure 2.5, the examined multilevel system of tank design is situated at Level 1, indicating that prediction models at all levels are used to describe the performance of the overall tank system. An increase in design level indicates an increase in design complexity described by a decrease in length scale used to predict overall tank performance. The first three levels illustrated relate to system specifications including the overall system (tank design), subsystems (BRP subsystem), and components (BRP core layer). Increasing in model complexity, material specifications at the microscale, nanoscale, and quantum levels are considered in modeling tank behavior, representing the multiscale materials design portion of this multilevel-multiscale design process.

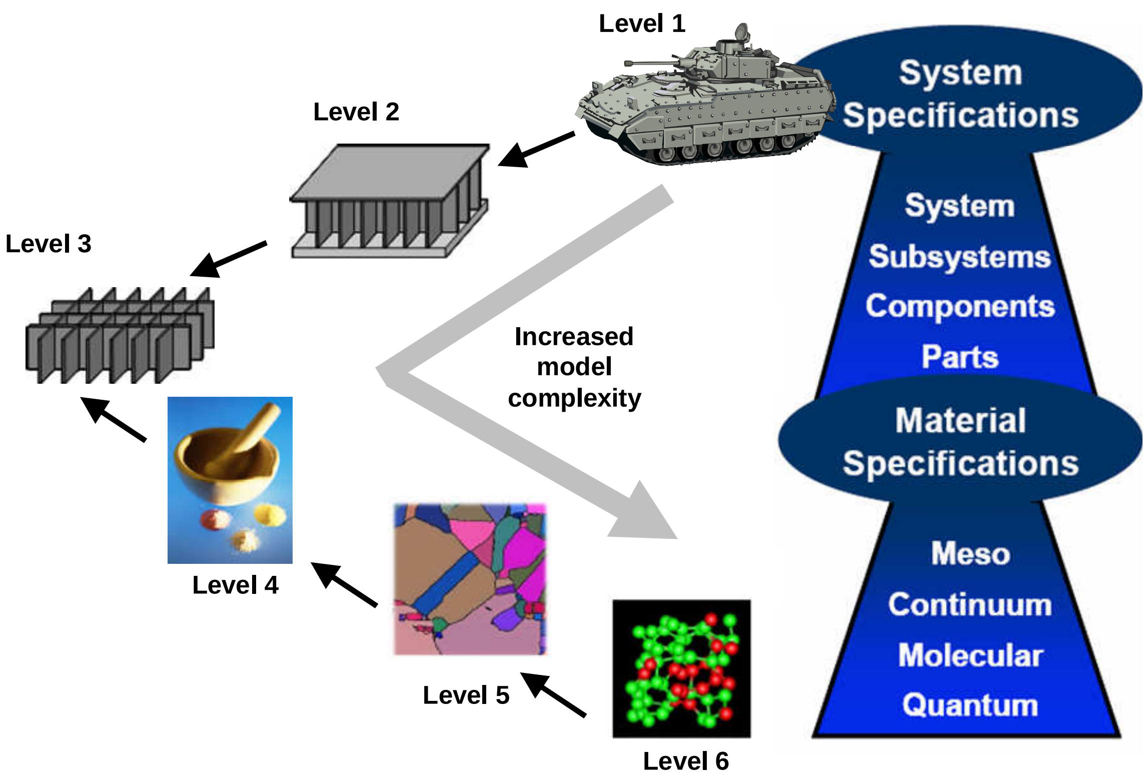


Figure 2.5 – Multilevel-multiscale design process

Multilevel design is also described using homogenization techniques categorized as multilevel-homogenization design. Multilevel-homogenization design is a multilevel design problem at a single length and / or time scale divided into levels of model complexity. In a multilevel-homogenization design problem, model complexity (or simplicity) is achieved by approximating system performance with varying degrees of accuracy. Simplified system performance models are derived based on approximations of more complex performance models through a series of homogenization techniques. BRP design described as a multilevel-homogenization process is shown in Figure 2.6.

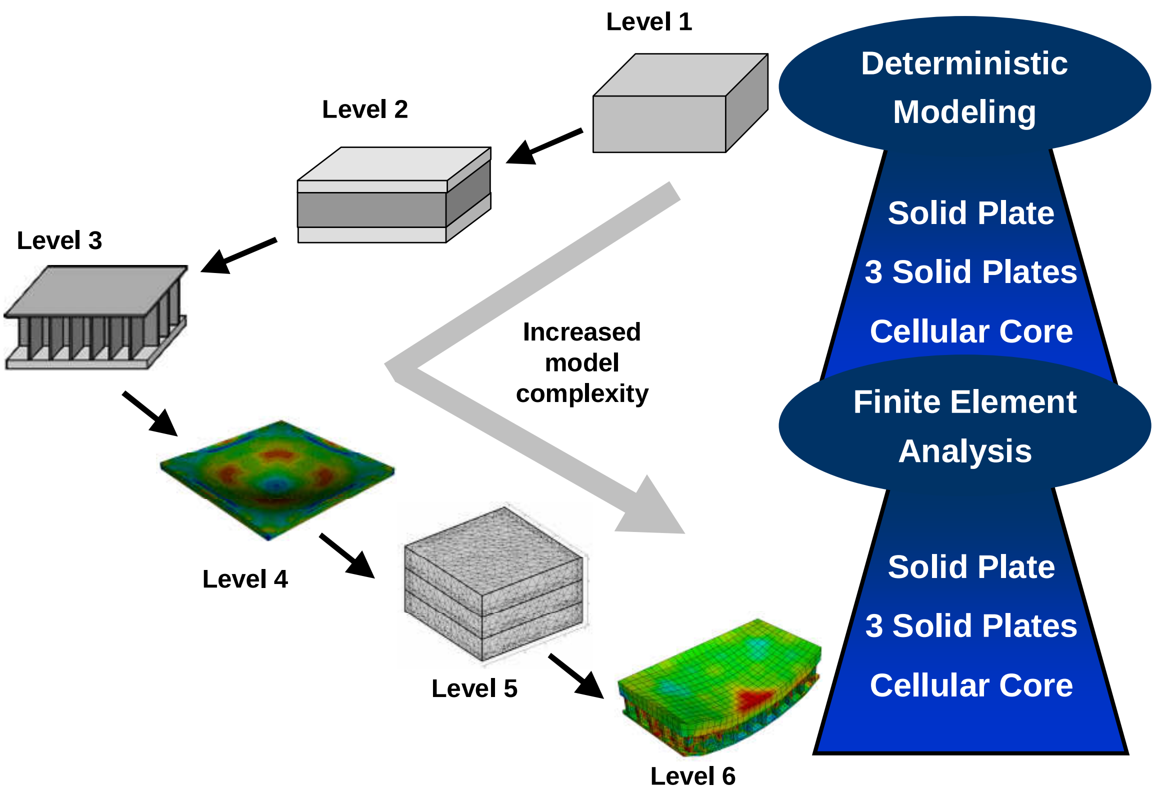


Figure 2.6 – Multilevel-homogenization design process

In Figure 2.6, the designed multilevel system is a BRP, shown at each level in the design process. In contrast to Figure 2.5, all levels of BRP design in Figure 2.6 relate to design at a single length scale. An increase in design level the multilevel-homogenization process in Figure 2.6 results in an increase in model complexity used to predict BRP performance. The multilevel-homogenization process in Figure 2.6 is divided into two segments—modeling BRP performance using deterministic models and modeling BRP performance using finite element analysis. The least complex level in Figure 2.6 represents modeling BRP performance using deterministic performance models of a single, solid plate. Increasing in model complexity, BRP performance is predicted using analytical models representing multiple panels and a honeycomb core. As BRP performance is more accurately modeled, finite element analysis is used to predict system performance. The most complex level in Figure 2.6 is finite element analysis of a three-layer BRP with honeycomb core used to model BRP performance.

Multilevel-multiscale and multilevel-homogenization design process are related, as shown in Figure 2.7.

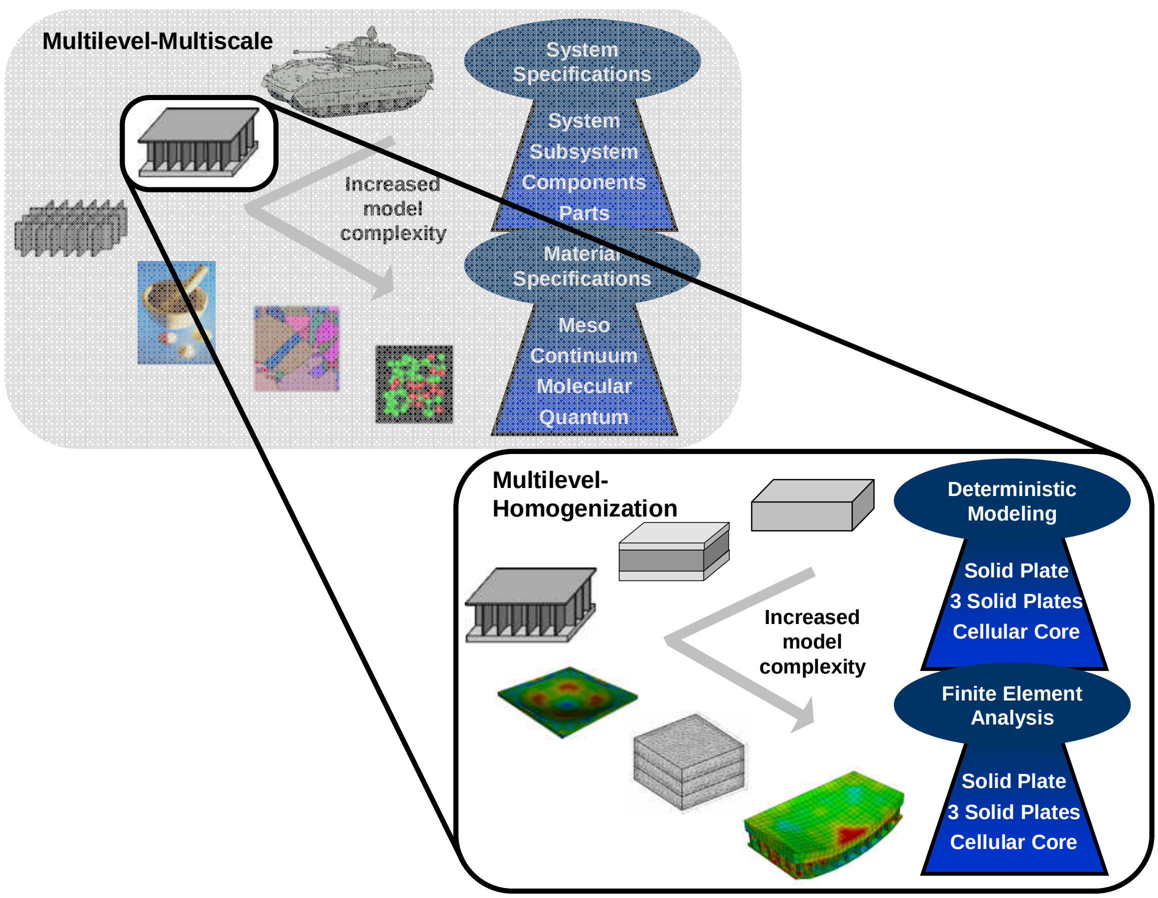


Figure 2.7 – Multilevel-multiscale vs. multilevel-homogenization

Multilevel-homogenization is a method for further investigating a single level of a multilevel-multiscale design process. Once a multilevel-multiscale design problem is established, a multilevel-homogenization approach is employed to make design decisions at a single level of multilevel-multiscale design. As seen in Figure 2.7, a multilevel-multiscale design process involving tank design is presented. Then, a multilevel-homogenization process, based on Level 3 BRP design is abstracted from the multilevel-multiscale design process. The multilevel-homogenization design process involving BRP design is an approach for modeling BRP behavior using various levels of model complexity. Performance models for BRP design from the multilevel-homogenization process are then inserted into the multilevel-multiscale design process in order to aid the designer in decision-making in the complex multilevel-multiscale design environment. The multilevel design template presented in Chapter 3 can be applied in either multilevel-multiscale or multilevel-homogenization design problems. Example problems investigated in this thesis are classified as multilevel-homogenization because design phenomena at a single length scale are modeled with varying degrees of accuracy using homogenization techniques.

2.2 UNCERTAINTY AND ROBUST DESIGN

In the following section, a discussion on the classification of uncertainty and techniques for managing uncertainty in a design process are presented. Information in Section 2.2 is leveraged with minor modifications from the Ph.D. dissertation of Hae-Jin Choi (Choi 2005). Section 2.2 on uncertainty and robust design is included in this thesis in order to provide a thorough background on robust design techniques, and identify opportunities for advancement in this area, specifically related to multilevel design. The template-based approach to multilevel robust design presented in Chapter 3 is rooted in the multilevel robust design method presented in the following section, and discussed in detail in Chapter 3.

2.2.1 Uncertainty in a Design Process

There are four sources of uncertainty that are associated with uncertainty embedded in system functions of an engineering design problem. Uncertainty sources include (a) non-parametric system noise, (b) un-configured system noise, (c) model parameter uncertainty, and (d) model structure uncertainty. First, uncertainty of system functions may arise from "*non-parametric system noise*". Non-parametric system noise is the source of noise that is difficult for designers to parameterize numerically. When system responses vary without the variance of noise factors or control factors, the system may include non-parametric system noise. However, non-parametric system noise is difficult to represent numerically, resulting in a challenging issue in system design at small length levels. Second, system functions could have "*un-configured system noise*". This un-configured system noise is similar to non-parametric system noise because system response varies without input changes. However, in this case, the system has un-configured parameters that could be parameterized as numeric forms, but are not because of limited knowledge of and/or data for the system. Since un-configured system noise can be reduced by increasing the knowledge of the system, it is categorized as model structural uncertainty.

2.2.2 Robust Design Definition

Robust design is a method of improving the quality of products by reducing sensitivity to uncertainty in noise factors, control factors (design variables), and models. When product designs are robust, performance levels remain stable despite the presence of noise factors (Taguchi 1986; Taguchi 1990). Various types of uncertainty are shown in Figure 2.8. Types of uncertainty include uncertainty in noise factors (Type I), uncertainty in control factors (Type II), uncertainty in system models (Type III), and propagated process chain uncertainty (multiscale uncertainty). Robust design techniques have been developed specifically for each type of uncertainty shown in Figure 2.8. A more thorough explanation of types of uncertainty and robust design techniques is given in Section 2.2.3.

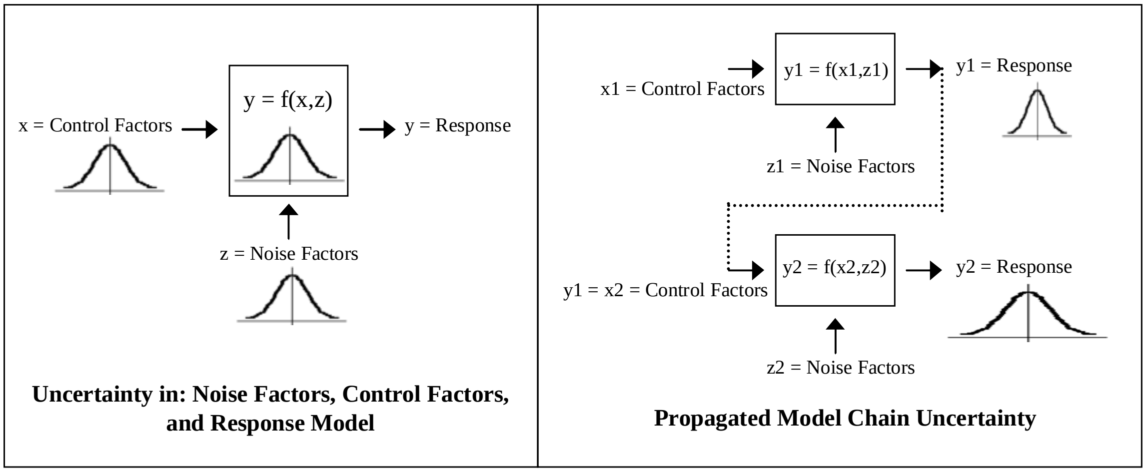


Figure 2.8 – Types of uncertainty in design processes

In typical robust design problems, the variance of a performance objective function is minimized with respect to environmental conditions, loading conditions, material properties, and other noise factors (Seepersad 2004). The variance of the response is minimized while also maximizing the response, minimizing the response, or bringing the response to a target. The robust solution may be inferior to the optimum solution in the absence of variation; however, the robust solution produces predictably satisfactory results in the presence of variation.

2.2.3 Robust Design Classification

The following section on uncertainty and robust design, unless otherwise noted, are leveraged with slight modifications from the work of Hae-Jin Choi in his Ph.D. dissertation (Choi 2005). Efforts in this thesis are focused on extending the implementation of multilevel robust design techniques, rather than the development of new methods to facilitate multilevel robust design. Therefore, the review of uncertainty and robust design completed by Choi provides a satisfactory review of these topics for the work presented in this thesis. In the following section, definition and classification of uncertainty are presented.

Type I Uncertainty

One of the main forms of uncertainty in a system model is uncertainty in uncontrollable independent system parameters, which are known as "*noise factors*". Noise factors are in parametric form and may be quantified and characterized as continuous numbers with or without probability information. Noise factors are usually given in system models as environmental factors, operating conditions, boundary conditions, or materials property variances that may be represented as continuous parameters and cannot be controlled by designers. Uncertainty in noise factors can exist as one of the aforementioned uncertainty types (Section 2.2.1); however, the most dominant type of uncertainty is variability (natural uncertainty), which can be measured using statistical methods. The degree of uncertainty in noise factors can be decreased by increasing the size of sampling and/or adapting efficient uncertainty analysis methods, leaving only irreducible statistical variability. In order to design a system robust to the uncertainty in noise factors, Type I robust design was proposed by Taguchi (Taguchi 1987).

Type I Robust Design: Identify control factor (design variable) values that satisfy a set of performance requirement targets despite variation in noise factors.

Type I robust design is used to design systems that satisfy a set of performance requirement targets despite variations in noise factors which are uncertain, uncontrollable, independent, system parameters. Although Taguchi's robust design principles (Taguchi 1993) are advocated widely in both industrial and academic settings, his statistical techniques, including orthogonal arrays and signal-to-noise ratio, have been criticized extensively, and improving the statistical methodology has been an active area of research (Chen, et al. 1996; Myers and Montgomery 1995; Nair 1992; Tsui 1992; Tsui 1996). During the past decade, a number of researchers have extended robust design methods for a variety of applications in engineering design (Cagan and Williams 1993; Chen, et al. 1996; Chen and Lewis 1999; Mavris, et al. 1999; Parkinson, et al. 1993; Su and Renaud 1997; Yu and Ishii 1994).

Type II Uncertainty

The second form of uncertainty in a system model is uncertainty in controllable system variables, which are known as "*control factors*". Similar to noise factors, control factors are also represented parametric form, measured and characterized as continuous numbers with or without probability distribution. Control factors are usually derived from the characterized parameters in system models that relate to system performances, including geometric information, mass, electrical, mechanical, or chemical inputs, amounts of constituents in materials, process control inputs, etc. Designers can determine the means of control factors; however, the deviations of control factors may not be controllable. Therefore, control factors should be characterized in a manner similar to noise factors. In order to design a system robust to the uncertainty in control factors, Type II robust design was proposed by Chen and coauthors (Chen, et al. 1996) as shown in Figure 2.9.

Type II Robust Design: Identify control factor (design variable) values that satisfy a set of performance requirement targets despite variation in control and noise factors.

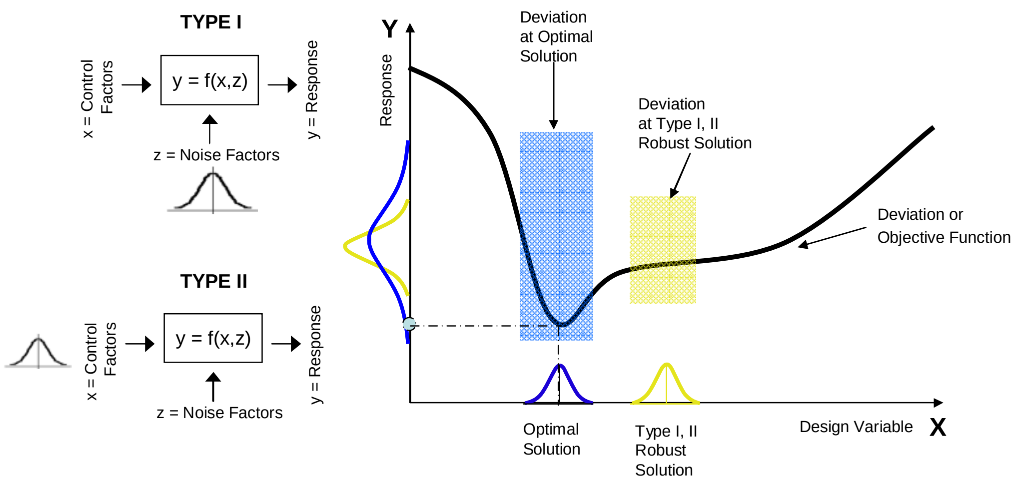


Figure 2.9 – Robust design for variations in noise factors and control factors. Modified from: (Choi 2005)

Type II robust design is used to design systems that are robust to possible variations in system parameters as a design evolves. In Type II robust design, designers search for means of control factors that satisfy a set of performance requirement targets despite variation in control factors. A method combining Types I and II robust design in the early stages of product development, namely, the Robust Concept Exploration Method (RCEM) has been developed (Chen 1995). RCEM is a domain-independent approach for generating robust, multidisciplinary design solutions. Robust solutions to multifunctional design problems are preference-weighted trade-offs between expected performance and sensitivity of performance due to deviations in design or uncontrollable variables. These solutions may not be absolute optima within the design space. By strategically employing experiment-based metamodels, some of the computational difficulties of performing probability-based robust design are alleviated. RCEM has been employed successfully for a simple structural problem and design of a solar powered irrigation system (Chen 1995), a High Speed Civil Transport (Chen, et al. 1996), a General Aviation Aircraft (Simpson, et al. 1996), product platforms (Simpson, et al. 2001), and other applications (Chen, et al. 2001).

Type III Uncertainty

The third factor for the uncertainty embedded in system functions is "*model parameter uncertainty,*" which is due to a combination of limited data and nonparametric system noise (or un-configured system noise). For example, if a nondeterministic system analysis is computationally intensive or experimentally expensive, then the limited data will result in uncertain parameters in metamodels (such as response surface models) of the system response. This is the typical type of uncertainty in materials design that employs computationally intensive models. The final factor for the uncertainty embedded in a system model is "*model structural uncertainty*" that is due to assumptions and idealization in a system. For example, model structural uncertainty includes linearization and discretization errors in finite element analysis, errors in computer codes, employment of uncertain knowledge, and other assumptions due to limited information. The uncertainty embedded in a system model cannot be managed by previous robust design approaches (Type I and II). In order to manage this uncertainty, a new type of robust design approach, called Type III robust design, is proposed. A visual representation of Type III robust design, compared to Type I and Type II robust design, is shown in Figure 2.10.

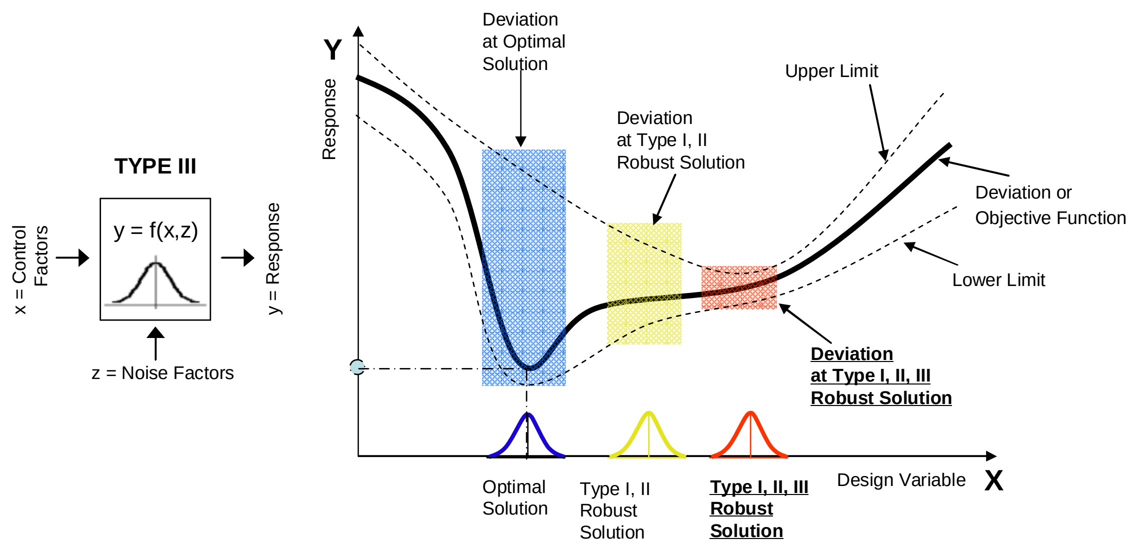


Figure 2.10 – Type III robust design (Choi 2005)

Type III Robust Design: Identify adjustable ranges for control factors (design variable), that satisfy a set of performance requirement targets and/or performance requirement ranges and are insensitive to the variability within the model.

Type III Robust Design is illustrated in Figure 2.10. In the figure, the same objective function curve is employed to show the differences among the optimal solution, Type I and II robust solution, and Type I, II and III robust solutions. A deviation (or objective) function, which represents the system's response, is illustrated as a solid curve. In addition, two dotted curves are added around the objective function, representing uncertainty limits, which is due to the non-parametric variability, un-configured variability, and model parameter uncertainty as mentioned above. Considering not only the objective function but also the two uncertainty limits, the optimal and Type I and II robust solution have larger performance deviations than the Type I, II, and III robust solution.

Type III robust design becomes more important since modern engineering systems are getting more and more complex (or extremely small) and their behaviors are stochastic. Compared to Type I and II robust design, Type III robust design has not been studied rigorously in engineering systems design. The absence of the studies is due to the ignorance of this uncertainty in most of traditional engineering systems design problems or the difficulties in quantifying and incorporating this uncertainty into a design exploration process.

For Type III robust design, it is required to build error bounds (uncertainty bounds) in a model in a computationally inexpensive manner. The most accurate way to incorporate the embedded uncertainties as well as the uncertainty in control and noise factors during design exploration is to perform actual simulation using statistical techniques (simulation-based design). Monte Carlo Simulation is a popular method to measure variations of performance by simulating input variations (uncertainty analysis). Du and coauthors employed this approach for a relatively simple problem (Du and Chen 2000).

Even though this approach could produce accurate results in design exploration, it requires a large number of experiments (more than 10,000 in many cases) for uncertainty analysis even in a single evaluation during a design exploration process. However, most multiscale material performance analyses need intensive computing power (from half an hour to several days for a single simulation run). It is nearly impossible to employ this approach in materials design exploration even if a sampling technique, such as Latin Hypercube sampling, is applied to reduce the number of experiments. Computationally inexpensive uncertainty analysis methods are needed to solve this problem. In this section, Type III robust design is defined. In the next section, a strategy for managing propagated uncertainty in a design/analysis process chain is presented.

Multiscale Uncertainty

The final type of uncertainty in a complex system model is that generated in the design and analysis process chain, which, unlike the aforementioned uncertainties in a system model, arises from the complex design and analysis process chain and not from the system model itself. This type of uncertainty is often observed in multidisciplinary uncertain system design problems and includes errors in decisions made by other designers and accumulated errors (propagated uncertainty) by subsequent series of uncertain subsystem models. Typically, complex multidisciplinary system design requires multiple experts to collaborate to make decisions for designing a system. The outputs of other experts' decisions in a subsystem could be input parameters, constraints, or design spaces of other subsystems or systems design. In many cases, multiple subsystem designs even share common design variables. In these interactions in design activity, a subsystem design error can be propagated to another subsystem or system. Additionally, complex systems design tends to employ multiple analyses and simulations in a series to predict system responses.

Multiscale Robust Design: Identify adjustable ranges of control factor (design variable) values under potential uncertainty and uncertainty propagation in a design and analysis process chain; account for uncertainty in downstream activities and uncertainty propagation.

Multiscale robust design is focused on uncertainty associated with the design process chain as shown in Figure 2.11.

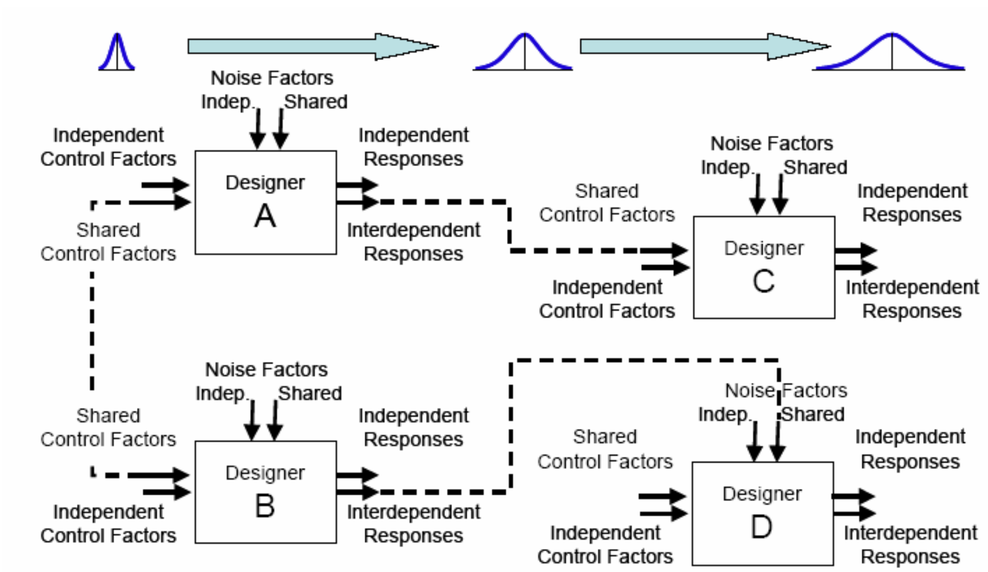


Figure 2.11 – Multiscale robust design

Design process uncertainty emanates from: (a) changes in design specifications as a result of downstream or concurrent decisions and design activities or (b) the propagation and potential amplification of uncertainty due to the combined effect of analysis tasks performed in series or in parallel. Both sources of design process uncertainty are common and important for multidisciplinary design and analysis, including multiscale, multi-physics materials design, with a plethora of shared or coupled variables and analyses performed on multiple length and time levels. The information dependency in multiscale models engenders complex design process chains – hierarchical, parallel, and serial design processes.

The underlying concepts in multiscale robust design provide the theoretical basis for a multilevel design template developed in Chapter 3 and implemented in the multilevel design problems in Chapter 4 and Chapter 5. In Chapter 3, it is shown how the concepts of multiscale design are adapted for a multilevel robust design method, the Inductive Design Exploration Method (IDEM) (Choi, et al. 2005). The usefulness of IDEM is extended by adapting the multiscale robust design method IDEM to a template-based approach to multilevel design. Details regarding research opportunities in multiscale robust design are given in Section 2.4.

2.3 DESIGN TEMPLATES

In Section 2.3, a discussion of a template-based approach to engineering design is presented. Section 2.3.1 begins with general information on using design templates as design process building blocks. Then, the key requirements of design templates are given in Section 2.3.2. Section 2.3.3 contains a detailed explanation of a specific design template implemented in this thesis—the Compromise Decision Support Problem (cDSP). The cDSP is a framework for solving nonlinear, multi-objective design problems and is utilized in the example problems in this thesis. The cDSP also provides the structural basis for the multilevel design template presented in Chapter 3.

2.3.1 Design Templates in Engineering Design

Design templates are a key topic addressed in this thesis. Recall from the glossary of key terms that a design template is a design process building block with a preset format, used as a starting point for a particular design application so that the format does not have to be recreated each time it is used (Panchal, et al. 2005). The main goal in this thesis is the development of a design template for facilitating a multilevel robust design process. A review of design templates is included in this thesis because of its direct link to one of the main research topics presented in this thesis. Section 2.3.1 on design templates is taken from the Ph.D. dissertation of Jitesh Panchal (Panchal 2005) with only slight modification. The concepts relating the design templates presented in Panchal's Ph.D. dissertation are extended in Section 2.3.2 to include requirements for design templates in engineering design.

One of the main challenges in modeling any design effort, regardless of level or scope, is standardizing the manner in which information and associated dependencies are represented. The need for reusability of information translates this requirement into representing information in a domain-neutral form that supports designers in providing and structuring required information content in a computationally archival and reusable manner. This calls for a domain-independent means of capturing design information. In order to facilitate designer interactions required for effective collaboration from a decision-based perspective, expression of design decision related information in a standardized format is also required. It is for this reason that a *modular template-based approach* to modeling design information is advocated. A *template* is commonly defined as (1) a pattern, used as a guide in making something accurately, (2) a document or file having a preset format, used as a starting point for a particular application so that the format does not have to be recreated each time it is used[2.3]. Clearly, the word *template* is appropriate in the context of this thesis because it implies reusability, achievability, and support/guidance.

In order to effectively support engineering design processes, this notion translates to the development of reusable computational templates for design. These computational templates should serve as building blocks – completely modular components that are standardized with respect to structure and interface architecture. Such building blocks must also facilitate analysis, and execution. Currently, there is a lack of formal, executable, computational models for representing and reusing existing knowledge about design processes. The only knowledge that is readily available is confined either to designers' expertise or to descriptive/pictorial forms of documentation. This is a result of the predominantly narrative or symbolic nature of current models.

2.3.2 Requirements for Design Templates

The key requirements of design templates are discussed in Section 2.3.2. These requirements describe the essence of a design template. Also, the following requirements detail the advantages of using a template-based approach to multilevel design. The descriptions provided in the following section are given in the context of template-based engineering design.

Reusable

Reusability relates to the ability to use a single system in multiple instances. In engineering design, reusability is relevant both within a particular design problem and among multiple design problems. Reusability describes one of most crucial requirements of design templates. One of the main purposes of creating design templates from existing

[2.3] Compiled from www.dictionary.com

design methods is to store a method's procedural information in a form that can be easily reused and applied to multiple design problems. Reusability in design templates is also useful at a lower level of design process abstraction. Design templates are often used to store information that is used multiple times in a single design process in a format and location that is easily accessible in all stages of a design process. For example, information relating the design goals and preferences is important at several stages in a design process. By using a design template to gather this information, the "design goals and preferences template" contains information that is readily available at all stages in design.

Modular

Modularity is the characteristic of "being composed of standardized units or sections for easy construction or flexible arrangement[2.4]." Also, a modular system is one which possesses the ability for a portion of the system to be altered without disturbing the remainder of the system. Modularity in a system is best illustrated by considering standardized building blocks (such as Legos®). Standardized building blocks can be joined to produce a variety of configurations to meet current design requirements. Similarly, design-templates should be modular, allowing for unique configuration to better meet the requirements of each design problem.

Mutable

Mutability (also called flexibility) is a characteristic of being available and capable of change. It is important for design templates to be flexible to change so that generic design templates can be particularized for individual design problems. In Chapter 4 and Chapter 5, the flexibility of the developed design template is demonstrated as it is particularized in solving two example problems. The characteristics of modularity and

[2.4] www.dictionary.com

mutability work together in the design of design templates. It is desirable that one can easily make changes to portions of a design template without negatively affecting the entire template. For example, it is desirable that the design constraints template be capable of adjustments without affecting the performance of other templates in a design process.

Archival

An archival system posses some inherent value such that it is stored for future reference, implementation, examination, and / or augmentation. One of the key advantages of design templates is that they can be used to store design process information. By implementing a design template in a design process, the designer stores information regarding the information and the flow of information throughout the design process. Following this procedure of continual and thorough data storage throughout a design process can become beneficial in design augmentation and design trouble-shooting, design. Additionally, storing design information in a standardized format encourages a collaborative environment in which information is shared among design stakeholders.

2.3.3 The Compromise Decision Support Problem – A Design Template

Based on the discussion on design templates in engineering design, the compromise Decision Support Problem, as it relates to design templates, is discussed in the following section. The compromise Decision Support Problem is implemented in the example problems included in this thesis. Therefore, it is appropriate to include a review of the compromise Decision Support Problem to provide a theoretical background for the method in which the example problems are solved. The following information on the compromise Decision Support Problem (cDSP) is leveraged with only minor modifications from the Master's Thesis of Andrew Schnell (Schnell 2006). The development and implementation of the cDSP is discussed in detail in many papers published by the Systems Realization Laboratory (SRL). In his Master's Thesis, Schnell provides a detailed summary of the cDSP, as described by former and current members of the SRL.

In this section, the work of the Systems Realization Laboratory towards creating a means to support design decision-making is summarized the Compromise Decision Support Problem (cDSP) is introduced. The cDSP is the backbone technology that facilitates the frameworks presented in the next sections. The purpose of this section is to review the literature regarding the use of Decision Support Problems in solving engineering design problems, to build confidence in the use of DSPs to solve engineering design problems in product design. The discussion begins with a brief introduction into the history of DSPs and specifically cDSPs. This section concludes with a discussion regarding how the cDSP has been extended to improve its usefulness, and the ease with which the problems can be formulated using reusable templates.

The Compromise Decision Support Problem (cDSP)

Decision Support Problems (DSPs) provide a means of modeling decisions encountered by a human designer. Multiple objectives that are quantified using analysis-based “hard” and insight-based “soft” information can be modeled in DSPs. Compromise and selection Decision Support Problems are two flavors of DSP. Selection DSPs serve as decision models for selecting between design alternatives. The compromise DSP is a decision model for solving multi-objective, non-linear, optimization problems (Mistree, et al. 1990). Mathematically, the cDSP is a multi-objective decision model which is a hybrid formulation based on Mathematical Programming and Goal Programming (Mistree, et al. 1993a). It is used to determine the values of design variables, which satisfy a set of constraints and bounds and achieve as closely as possible a set of conflicting goals. The word formulation of the cDSP is shown in Figure 2.12. The mathematical formulation of the cDSP is shown in Figure 2.13.

Given:	A feasible alternative, assumptions, parameter values, and goals
Find:	Values of design and deviation variables
Satisfy:	System constraints, system goals, and bounds on variables
Minimize:	A deviation function (deviation variables that measure distance between goals targets and design points)

Figure 2.12 – Word formulation of cDSP (Mistree, et al. 1993a)

GIVEN

An alternative to be improved through modification
Assumptions used to model the domain of interest
The system parameters:

n number of system variables
p+q number of system constraints
p equality constraints
q inequality constraints
m number of system goals

$C_i(\mathbf{X})$ Capability of the system
$D_i(\mathbf{X})$ Demand to the system
$g_i(\mathbf{X})$ System constraint function
$g_i(\mathbf{X}) = C_i(\mathbf{X}) - D_i(\mathbf{X})$
$f_k(d_i^+, d_i^-)$ Function of deviation variables to be minimized at priority level k the preemptive case

FIND

X_i System Variables i = 1,..., n
d_i^+, d_i^- Deviation Variables i = 1,..., m

SATISFY

System Constraints (linear, nonlinear)
$g_i(\mathbf{X}) = 0$ i = 1,..., p
$g_i(\mathbf{X}) \geq 0$ i = p+1,..., p+q

System Goals (linear, nonlinear)
$A_i(X) + d_i^- - d_i^+ = G_i$ i = 1,..., m

Bounds
$X_{i,\min} \leq X_i \leq X_{i,\max}$ i = 1,..., n
$d_i^+, d_i^- \geq 0$ i = 1,..., m
$d_i^+ \cdot d_i^- = 0$ i = 1,..., m

MINIMIZE

Deviation function: Archimedean formulation
$Z = \sum_i W_i (d_i^+, d_i^-)$ i = 1,..., m

Figure 2.13 – Mathematical formulation of cDSP (Mistree, et al. 1993b)

Currently, there are two types of deviation function used in formulating a compromise DSP: the Archimedean solution scheme and the Preemptive approach (Ignizio 1985). In this thesis, the Archimedean approach is used exclusively, therefore the Preemptive approach is not included in the mathematical formulation of the cDSP in Figure 2.10. In the Archimedean approach, the deviation function, *Z*, is simply a weighted sum of the deviation variables of each of the objectives.

A solution to a compromise DSP is called a satisficing solution. "Satisficing" is a term coined in the context of mathematical optimization, meaning not the best but good enough (Simon 1996). In a compromise DSP, the bounds and constraints form the feasible design space. The solution of the compromise DSP is a point selected within the feasible design space based on its degree of satisfaction to a set of conflicting design goals. Satisfaction is evaluated using the value of the deviation function in the compromise DSP. The human designer or designers must decide whether the solution of the compromise DSP is acceptable or further investigations should be conducted by modifying the aspirations and/or the feasible design space.

The values of the deviation variables (d_i^- d_i^+) indicate the extent to which each of the goals have not been achieved and thus are a source of useful information (Simpson 1998). The deviation variables represent the levels of overachievement and underachievement, respectively, of a goal. The deviation variables are never negative, and one of them will always be zero. This rule is represented by Equation 2.1.

$$\begin{aligned} &d_i^+ \geq 0 \\ &d_i^- \geq 0 \\ &d_i^+ \cdot d_i^- = 0 \end{aligned} \tag{2.1}$$

Robust Design and the Compromise Decision Support Problem

As discussed in Section 2.2, a robust design solution is one which is insensitive to variation. That is, a robust design will have predictable performance despite slight variation in input parameters. The cDSP can be used in robust product design by setting a design goal to include the minimization of response variation. Therefore, in robust design using the cDSP it is necessary to recognize at least two design goals: (1) minimize

/ maximize / bring to target performance, (2) minimize performance variation. The challenge exists in characterizing variation in a design process. In the following section, a discussed on methods for describing the impact of input uncertainty on performance variation are discussed. Information in the remainder of Section 2.3 is taken with slight modification from the Ph.D. dissertation of Carolyn Conner Seepersad (Seepersad 2004).

There are many techniques for transmitting or propagating variation from input factors to responses, and each technique has strengths and limitations. Monte Carlo analysis is a simulation-based approach that requires a very large number of experiments (Liu 2001). It is typically very accurate for approximating the distribution of a response, *provided* that probability distributions are available for the input factors. On the other hand, it is very computationally expensive, especially if there are large numbers of variables or if computationally expensive simulations are needed for evaluating each experimental data point. If only a moderate number of experimental points are computationally affordable, a variety of space-filling experimental designs are available such as Latin Hypercube designs (Koehler and Owen 1996; McKay, et al. 1979). If only a few experimental points can be afforded, sparse experimental designs such as fractional factorials or orthogonal arrays are available [e.g., (Myers and Montgomery 1995)]. Whereas, these experimental designs require fewer experimental points, they do not provide approximations of the distribution of a response, but they do provide estimates of the range(s) of response(s). All of these experimental techniques can be used in two ways: (1) to provide estimates of the variation or distribution in responses at a particular design point or (2) to construct surrogate models of the response that can then be used in place of a computationally expensive simulation model for evaluating mean responses and variations (Chen, et al. 1996; Mavris, et al. 1999; Welch, et al. 1990). They all suffer from the problem of size identified by Koch and coauthors (Koch, et al. 1999) in which the number of experiments becomes prohibitively large (given the computational expense of most engineering simulations) as the number of input factors or design variables increases. This characteristic is very important for multilevel design applications in which there are large numbers of variables and non-negligible computational requirements. Although experiments may be appropriate for evaluating the impact of variation of noise factors (which may be relatively few in number), they are not likely to be computationally attractive for evaluating the impact of variation in factors such as local material properties or design variables.

An alternative means for propagating variation is by Taylor series expansion (Phadke 1989). A first order Taylor series expansion, for example, can be used to relate variation in response, Δy, to variation in a noise factor, Δz, or a control factor, Δx, as follows:

$$\Delta y = \sum_{i=1}^{k} \left| \frac{\partial f}{\partial x_i} \Delta x_i \right| + \sum_{i=1}^{m} \left| \frac{\partial f}{\partial z_i} \Delta z_i \right| \tag{2.2}$$

where the variation could represent a tolerance range or a multiple of the standard deviation. Higher order Taylor series expansions can also be formulated to provide better approximation of the variation in response, but higher order expansions also require high order partial derivatives of the response function with respect to control and noise factors. Taylor series expansions are relatively accurate for small magnitudes of variation in control or noise factors but lose their accuracy for large variations or highly nonlinear function, *f*. A Taylor series expansion requires evaluation of the partial derivative or *sensitivity* of the response function with respect to changes in control or noise factors. If analytical expressions are available for the sensitivities, this can be a computationally attractive and relatively accurate approach (Bisgaard and Ankenman 1995), even for large numbers of control and noise factors. Otherwise, the sensitivities can be estimated using finite differencing techniques, automatic differentiation (a feature built into some computer programming languages), and other advanced techniques such as perturbation analysis and likelihood ratio methods (Andradottir 1998), but these techniques can diminish the computational attractiveness and accuracy of the approach. Sensitivity-based approaches have been proposed for modeling constraints (Parkinson, et al. 1993; Phadke 1989) and objectives (Belegundu and Zhang 1992; Su and Renaud 1997) in robust design. When analytical expressions are available, a Taylor series expansion is promising for propagating variation in control factors in the identification of multilevel design requirements. In this thesis, a first order Taylor's series expansion is used to model system performance variation. The Taylor's series method is selected due to its ease of use for the given example problems in Chapter 4 and Chapter 5 of this thesis.

Closing Thoughts on Template-Based Robust Design

Recall that Section 2.3 begins with a definition of design templates and their advantages in solving engineering design problems. In current design research, it is identified that design templates should be reusable, modular, mutable, and archival in order to add value to a design process. Next in Section 2.3, a previously developed design template, the cDSP, is discussed. The cDSP is recognized as a design template because it is a pattern for solving multi-objective, non-linear optimization problems. The cDSP is formulated at a high enough level of abstraction so that it can be applied to a variety of design problems without reconfiguration. At the end of Section 2.3, an approach for infusing robust design techniques in the cDSP is presented.

A discussion of the current research trends in template-based design is necessary because the main purpose in this thesis is the development of a template-based approach to multilevel robust design problems. The multilevel design template presented in Chapter 3 is based on the structure and information flow in the cDSP. Additionally, the cDSP is used when making design decisions at a single level in the example problems in Chapter 4 and Chapter 5. A thorough investigation of current design template research is intended to lead to research opportunities for advancement in this field. In Section 2.4 a research gap as related to template-based design is presented. The research questions for this thesis are developed in order to address portions of the identified knowledge gap in template-based design.

2.4 RESEARCH GAPS IN TEMPLATE-BASED MULTILEVEL DESIGN

The primary purpose of the literature review in Section 2.1 – Section 2.3 is to identify knowledge gaps relating to the key concepts of this thesis, a template-based approach to multilevel robust design. Ultimately, research in the area of template-based multilevel design is aimed at achieving a detailed multilevel design method and associated multilevel design templates. Although numerous detailed design methods for single level design have been developed and implemented (see Pahl and Beitz 1996) a detailed multilevel design process that addresses the critical needs of multilevel design (see Section 2.1.2 and Table 2.2) does not exist. The knowledge gaps identified in this thesis are intended to move the design community one step closer to a detailed multilevel design process composed of reusable, modular design templates. Figure 2.11 is a representation of the research gaps identified in this thesis, and the ultimate goal of multilevel template-based design research.

In Section 2.4 knowledge gaps relating to multilevel robust design and design templates are presented. The information in Section 2.4 is organized based on the two foundational concepts of this thesis, multilevel robust design and design templates. The research questions in Chapter 1 are developed in order to address the identified research gaps.

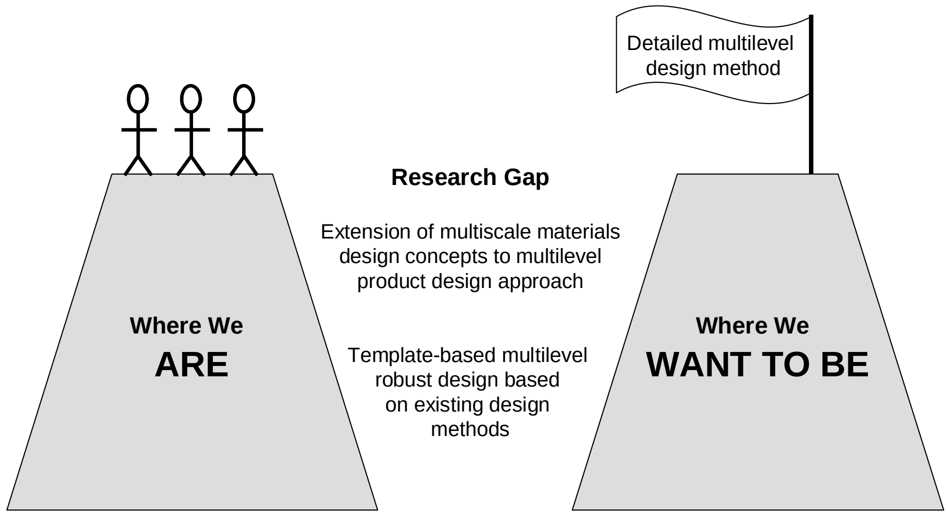


Figure 2.14 – Research gap in multilevel, template-based design

2.4.1 Research Gap Relating to Multilevel Robust Design – From Multiscale Design to Multilevel Design

In Section 2.1, the definitions of multilevel design and materials design as a multilevel design process are given. The critical needs of a multilevel design process compared to design at a single level are listed, and a requirements list for a multilevel design process is developed based on these critical needs. Section 2.2 contains a classification of four types of uncertainty in a design process. Robust design techniques addressing each type of design uncertainty are given. In Section 2.4.1 research gaps relating to multilevel design and multilevel robust design are given.

As stated in Section 2.1, much of the current research in multilevel design relates to material modeling and design, often referred to as multiscale materials design. The 'scales' in multiscale materials design denote various length and / or time intervals at which material prediction models are created and design decisions are made. The key advantage of a multiscale material modeling and design approach is that unmanageably complex materials design problems can be divided into segments capable of being processed by current computational tools. Information at each material scale is then combined in order to design an overall material system.

Based on a review and analysis of multiscale design literature, it is determined that an area of research opportunity involves extending the concepts of multiscale modeling and design beyond the materials design community. The concepts of multiscale materials design can be extended to include all forms of complex engineering design problems. In this thesis, multiscale material modeling and design techniques are adapted in a more general multilevel design approach. The term multilevel design represents a design process in which a complex design problem is divided and analyzed according levels of model complexity. Multilevel design is a more general case of multiscale materials design. In multilevel design, design levels represent the complexity of system prediction models and based on the number of design variables used in prediction models. Exploring a multilevel design approach is a key component of both research questions presented in Chapter 1. In Chapter 3, an approach for multilevel design using design templates is given. In Chapter 4 and Chapter 5, two multilevel design problems are solved using the developed multilevel design approach.

2.4.2 Research Gap Relating to a Template-Based Approach to Multilevel Robust Design

In Section 2.2 a discussion of design uncertainty and robust design techniques is given. In this thesis, there is particular interest in the multiscale robust design method (IDEM). The motivation for developing IDEM originally came from complex, multidisciplinary materials design problems, but IDEM can be applied to any design problem in which uncertainty is introduced due to design and analysis process chains (Choi 2005). In its current state, IDEM is a design strategy consisting of a series of steps and equations that, when applied, lead to a range of feasible multiscale robust design solutions. A research opportunity exists in combining the procedural steps of IDEM into the reusable, modular, mutable, and archival form of a design template.

Recall from Section 2.3 in which the topic of design templates is discussed. Template-based design is an engineering design strategy in which a generic design pattern built to aid in design decision-making is applied to a variety of design problems without changing its basic structure. A specific design template, the cDSP, is used in solving multi-objective, nonlinear design problems at a single level. The key research gap identified in this thesis is the potential advantage of combining the notion of template-based design with previously developed robust design methods, specifically IDEM. It is also identified that the cDSP, a design template for single scale analysis, can be extended to encompass multilevel design processes. The key research contribution in this thesis is to transfer the information in a multiscale robust design method (IDEM) into a multilevel design template based on the cDSP. The identified research gaps are addressed in the research questions in Chapter 1.

2.5 SYNOPSIS OF CHAPTER 2

Chapter 2 contains a review of multilevel design, robust design, and template-based design in order to identify research opportunities in these areas. The key research gaps identified in Chapter 2 are: the opportunity to expand a traditional multiscale design approach beyond the materials design community, resulting in multilevel design strategy for complex engineering design; and to adapt an existing multiscale robust design method to a design template based on the cDSP. The research questions presented in Chapter 1 are intended to address the research gap identified in Chapter 2.

Chapters 1 and 2 provide the motivation and frame of reference for the remainder of the thesis. Key topics addressed in this thesis—multilevel design, robust design, and template-based design—are discussed in detail in Chapters 1 and 2. Chapter 3 contains the theoretical foundation for the development of a design template to support multilevel robust design. In Chapters 4 and 5 the multilevel design template is applied to two example problems. Chapter 6 contains a detailed look at the verification of the multilevel design template as well as research opportunities in multilevel robust template-based design.

CHAPTER 3

THEORETICAL FOUNDATIONS FOR A MULTILEVEL DESIGN TEMPLATE

The information presented in Chapter 3 provides the theoretical foundation for the remainder of this thesis. In this chapter, the generic formulation of a multilevel design template is presented. The theory supporting the development of the multilevel design template comes from the compromise Decision Support Problem (cDSP) (Section 2.3), a previously developed multilevel robust design method (the Inductive Design Exploration Method [IDEM]) (Choi, et al. 2005), and a template-based design approach (Section 2.3). The usefulness of IDEM and the cDSP are extended by combining key concepts from each of these design tools and adapting them for a template-based design environment.

Chapter 3 begins with a discussion of IDEM as a base method for a multilevel design template. The steps of IDEM are described in detail. The overall usefulness of the base method and its assumptions are discussed. After the base method is sufficiently established, a discussion of building a design template from a design method is given. Specifically, the steps required to extend the base method into a reusable, modular, design template are presented. Next, the developed multilevel design template is presented and discussed. The overall usefulness and the underlying assumptions of the developed design template are given. The generic multilevel design template is particularized for application in the example problems presented in Chapter 4 (design of a cantilever beam and its associated material) and Chapter 5 (design of a BRP). Finally, the verification and validation of the multilevel design template is presented by examining its domain-independent structural validity and domain-independent performance validity. A summary of the information in Chapter 3 is given in Table 3.1 and Figure 3.1 illustrates how Chapter 3 is connected to other ideas in this thesis.

Table 3.1 – Summary of Chapter 3

Heading / Sub-Heading	**Information**
Multilevel Robust Design Base Method	
Overview of Base Method	Base method: - Robust design approach to multilevel design problems - Inductive Design Exploration Method (IDEM)
Base Method – Assumptions and Usefulness	Characteristics of IDEM: - Assumes that inductive multilevel design solutions cannot be directly calculated, but are found using a series of deductive mapping functions - IDEM used to produce multilevel design solutions that are robust to model structure uncertainty and process chain uncertainty
Base Method Procedure	Three procedural steps: - Define design levels and feasible space at each level - Map discrete points to solution ranges through deductive mappings (specific-to-general) - Map solution ranges to robust solution using developed deductive mapping functions in an inductive manner (general-to-specific)
Multilevel Robust Design Template	
From IDEM to a Multilevel Robust Design Template	Developing a design template: - Determine key steps in IDEM - Transform procedural information from IDEM to a modular, reusable, archival form
Formulation of Multilevel Robust Design Template	Multilevel robust design template: - Multilevel design template is based on the cDSP - Generic design template at a single design level is developed - Single level design templates are joined creating a multilevel design template
Multilevel Design Template Particularized for Examples	Multilevel design template is particularized for example problems discussed later in thesis: - Design of a cantilever beam (Chapter 4) - Design of a blast resistant panel (Chapter 5)
Verification and Validation	
Domain-Independent Structural Validity	Internal consistency of multilevel design template: - Template is based on an existing method, IDEM, with proven internal consistency - Template is successfully applied to two example problems indicating its internal consistency
Domain-Independent Performance Validity	Ability to apply template to a variety of multilevel design problems, not previously tested: - Template is sufficiently generic and can be particularized for a variety of design problems - Template is flexible and modular and can be adjusted for special needs in individual design problems
Thesis Roadmap	

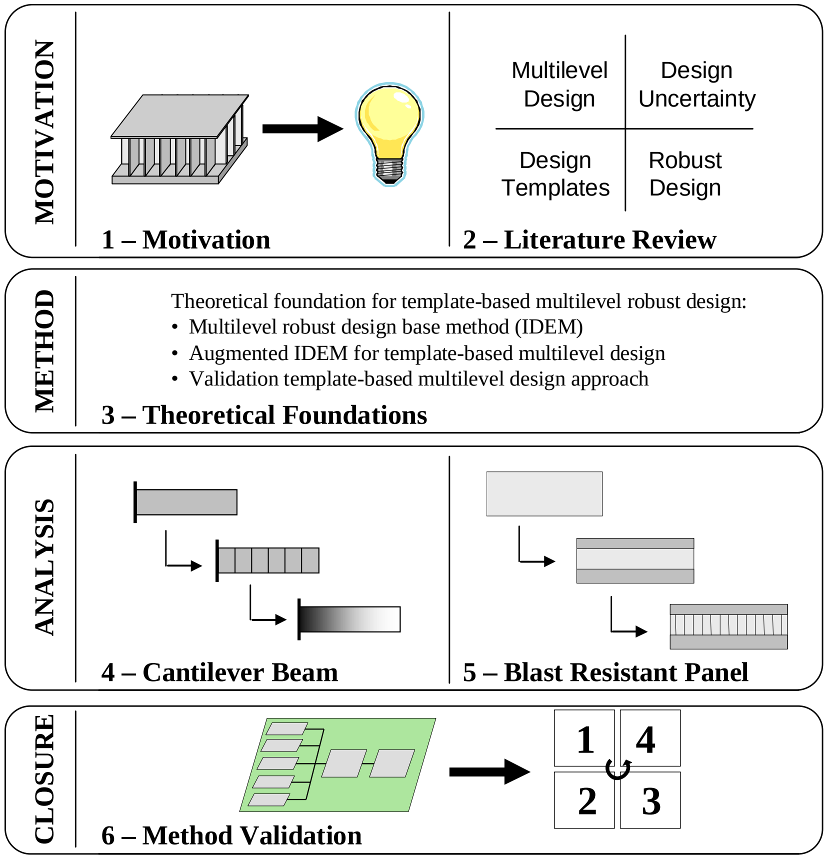


Figure 3.1 – Setting the context for Chapter 3

3.1 MULTILEVEL ROBUST DESIGN BASE METHOD

The main focus in this thesis is to develop a design template to facilitate multilevel robust design solutions (Research Question #1, Section 1.3). In order to address this challenge, it is decided that the multilevel design template should be based on an existing method for facilitating robust solutions to multilevel design problems. The Inductive Design Exploration Method (IDEM) is selected as the base method for this thesis because the method is designed to produce robust solutions to multilevel design problems (Choi, et al. 2005). In the following section, an overview of IDEM as a tool for generating multilevel design solutions is given. An overview of IDEM is given in Section 3.1.1. The overall usefulness and assumptions associated with IDEM are given in Section 3.1.2. IDEM is composed of three procedural steps, which are discussed in detail in Section 3.1.3.

3.1.1 Overview of Base Method

IDEM was developed by Hae-Jin Choi (Choi, et al. 2005) and is presented in his Ph.D. dissertation (Choi 2005). As stated in Section 2.3.3, multilevel robust design techniques are used to manage uncertainty associated with the design process chain. Design process uncertainty emanates from: (a) changes in design specifications as a result of downstream or concurrent decisions and design activities (model structure uncertainty) or (b) the propagation and potential amplification of uncertainty due to the combined effect of analysis tasks performed in series or in parallel (process chain uncertainty) (Choi 2005). An example of information flow in a model chain is presented in Figure 3.2. In Figure 3.2, suppose that design input parameters x1 and x2 have some associated uncertainty. Uncertainty in input parameters x1 and x2 lead to uncertainty in output responses y1 and y2. As shown in the figure, y1 and y2 combine to form the input parameter to f3. The combination of uncertainty responses y1 and y2 produces a value with greater uncertainty than what is observed in input parameters x1 and x2. Therefore, the resulting output parameter, z, contains increased uncertainty due to the combination of uncertain input factors, y1 and y2. In a similar manner, combining process chains in a multilevel design problem can lead to unexpectedly large variation in final output performance measurements.

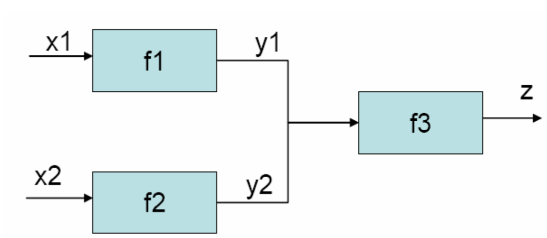


Figure 3.2 – An example of information flow in a model chain (Choi 2005)

3.1.2 Base Method – Assumptions and Usefulness

Basic assumptions and the general usefulness of IDEM are presented in the following section. Recall from Section 2.2.3 that design solutions from implementing IDEM "result in ranged sets that are robust to propagated and expanded uncertainty in a process and to the unquantifiable potential uncertainty a model might have" (Choi 2005 [Section 4.2]). The basic procedure of IDEM applied to multilevel aircraft design, is shown in Figure 3.3, Step 1 – Step 3.

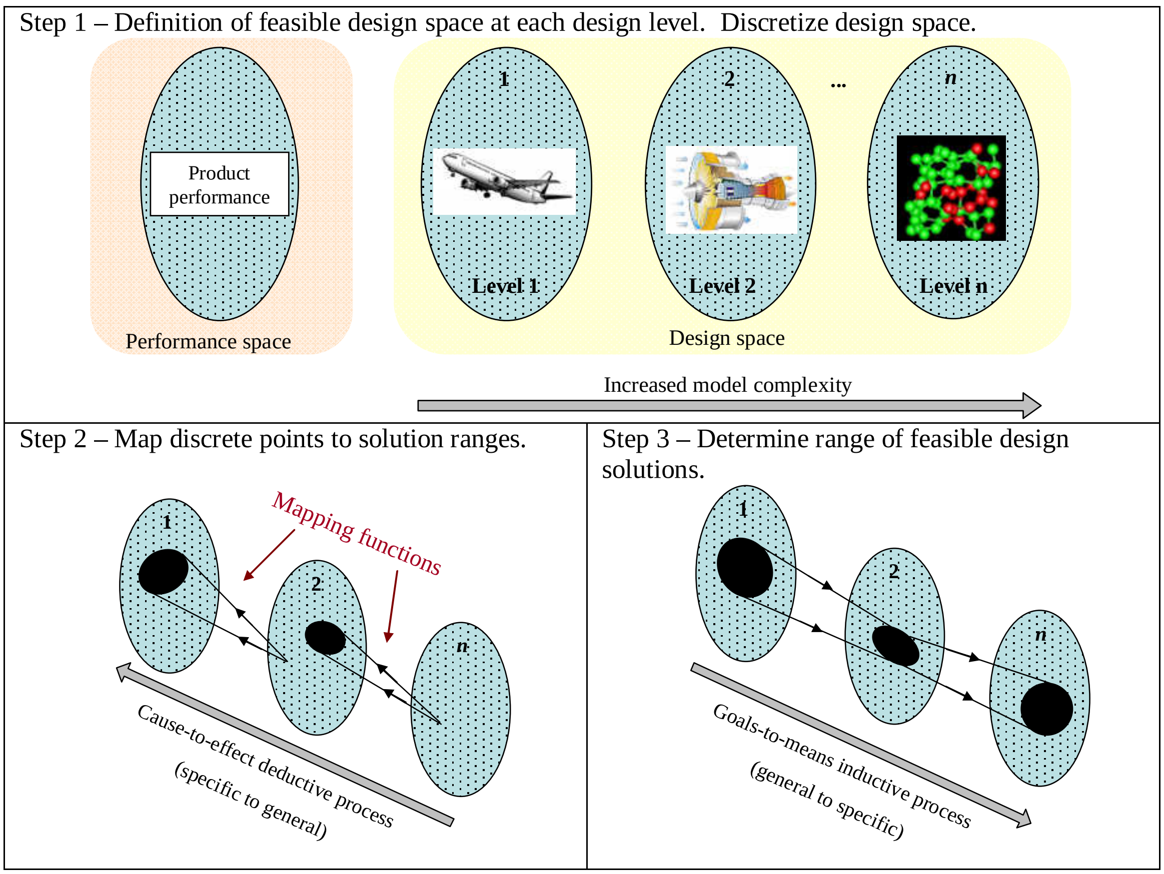


Figure 3.3 – Diagram of the inductive design exploration concept

First, the feasible design space at each level is determined (Step 1 in Figure 3.3). Then, mapping functions are used to map discrete points at Level n to solution ranges at Level n+1 (Step 2 in Figure 3.3). This mapping procedure is carried out in a deductive manner.

Finally, feasible solution ranges are determined at each level of the design by following an inductive design decision path based on previously developed deductive mapping functions (Step 3 in Figure 3.3). The final solution range is selected such that design freedom is preserved. A more detailed description of the base method procedure is found in Section 3.1.3. IDEM is used to inductively determine ranged sets of feasible solutions in a multilevel design problem and can be applied to a range of multilevel design problems. The solutions obtained from applying IDEM are robust to model structure uncertainty and process chain uncertainty. The final step of IDEM includes a method for strategically selecting a design solution within feasible solution ranges either emphasizing product performance or product robustness (Choi 2005).

IDEM has several underlying assumptions. The most crucial assumption implicitly stated in IDEM is that inductive solutions to multilevel design problems cannot be directly calculated. Evidence of this assumption in IDEM is that discrete points are mapped to solution ranges (deductive) before an inductive range of design solutions is reached. Also, in applying IDEM, the designer assumes that design levels are clearly defined and mapping functions can be created to accurately transfer information among design levels. An additional assumption of IDEM is that the design space is small enough to allow for an exhaustive search of all possible design combinations. For design problems containing approximately 10 or more design variables, characterizing the entire design space can be unrealistic. For such design problems, modifications to the base method, such as reducing the number of design variables, using metamodeling techniques, or employing parallel function evaluation, should be considered (Choi 2005).

3.1.3 Base Method Procedure

A procedure for multilevel robust design abstracted from IDEM is presented below in Step 1 – Step 3 and in Figure 3.4 (Choi, et al. 2005).

Procedure for multilevel robust design
Step 1: Define design levels in multilevel design process and feasible design space at each level
Step 2: Deductively map points to solution ranges
Step 3: Inductively determine range of feasible design

Figure 3.4 – Procedure for IDEM

The goal in this thesis is not to simply solve a design problem using IDEM, but to adapt the multilevel robust design method to a template-based design environment. Therefore, the exact details of IDEM are not discussed in detail. An overview of IDEM is given to show how it is used to inspire a template-based approach to multilevel robust design. For a complete description of IDEM, refer to the Ph. D. dissertation of Hae-Jin Choi (Choi 2005 [Chapter 4]).

Step 1: Define Design Levels and Feasible Design Space

To begin, a multilevel design problem is divided according to design levels. The process of partitioning a multilevel design problem into smaller units is unique for each design problem. However, several basic guidelines can be applied. The first challenge is to determine the basis from which to measure a design level. In traditional materials design applications, multiscale product and materials design is divided according to length scale (nano, micro, meso, meter, etc.). The main distinction between each scale involves a change in the overall length interval considered in a design process. However, in the research presented in this thesis, it is asserted that a multilevel design problem can be divided into levels according to a variety of characteristics. For example, the comprehensive example problem in this thesis involves the multilevel design of a BRP. The BRP design problem can be divided into levels according to length measurements (micro, meso, meter), reaction time measurements (millisecond, second, month, decade), or levels of model complexity (11 design variables, 9 design variables, 3 design variables). (The term multiscale often refers to materials design. The term multilevel is used in this thesis to denote a hierarchical product or process defined by changes in length, time, complexity, etc.). In this thesis, characterizing levels in terms of modeling complexity is investigated in solving the example problems. The metric a designer should use to partition a multilevel design problem should come from the natural behavior of the design problem. For example, if a multilevel design problem is most heavily dependent on reaction as a function of time, the various levels should be defined according to time levels.

Once the metric for partitioning a multilevel design problem is identified, various levels are defined. The process of determining level boundaries is unique for each design problem; however, several guidelines can be followed. First, a natural level boundary is identified at locations where it becomes necessary to alter design analysis tools. For example, in a multilevel materials design problem, different performance analysis tools are used when designing at the nano-level compared to the micro-level. A necessary change in design analysis tools provides a natural break in levels of a multilevel design problem. Additionally, it is important that an appropriate number of design levels are defined in order to accurately capture design phenomena without over-complicating the design problem. Partitioning the design problem into too few levels may result in the inability to capture certain relevant design phenomena, whereas defining too many design levels may result in an increase in design cost without a noticeable gain in design knowledge.

Once levels have been defined in a multilevel design problem, feasible design space at each level is specified. The feasible design space is identified by bounds placed on design variables. Since the design variables may be different at each level in a multilevel design problem, the feasible design space should be determined at each level. It is observed that all points in the feasible design space do not result in a design performance that is within given performance constraints. Therefore, mapping functions are developed to describe the deductive mappings from design levels to solution ranges. Once deductive mapping functions are developed (Step 2), solution finding algorithms are used to determine which points in the feasible design space result in design performance that is within specified performance constraints.

Step 2: Deductively Map Points to Solution Ranges

Mapping functions are created to translate information among levels. Due to the challenges of a multilevel robust design problem, deterministic mapping functions can only be created for a deductive transition among levels (moving from specific to general). Design information captured in mapping functions relates to design variable transformation and uncertainty propagation models. Design variable mapping models are used to describe how design information at a more specific design level maps into design information at a more general level. For the purposes of the example problems investigated in this these, variable mapping functions are used to describe the relationship of design variables among various levels of model complexity.

Uncertainty mapping functions are used to model uncertainty at each level of the design process, and the propagation of uncertainty throughout the design process. Uncertainty mapping functions are developed only when uncertainty in a design process can be modeled. In the example problems included in this thesis, it is assumed that design uncertainty is modeled by a known function. It is also assumed that uncertainty propagation among various levels is known and modeled using a specific equation or set of equations.

Step 3: Inductively Determine Range of Feasible Design Solutions

In the final step of IDEM, a robust design solution is presented as a range of possible solutions, obtained by inductive design space mapping. In the original method, developed by Choi (Choi 2005), a robust solution range is obtained by evaluating a metric for determining if a discrete point from an input space maps to a feasible design solution in the output space. This metric, called hyper-dimensional error margin indices (HD-EMI) is also used in determining a robust solution range. See Figure 3.5 for a visual and mathematical representation of HD-EMI.

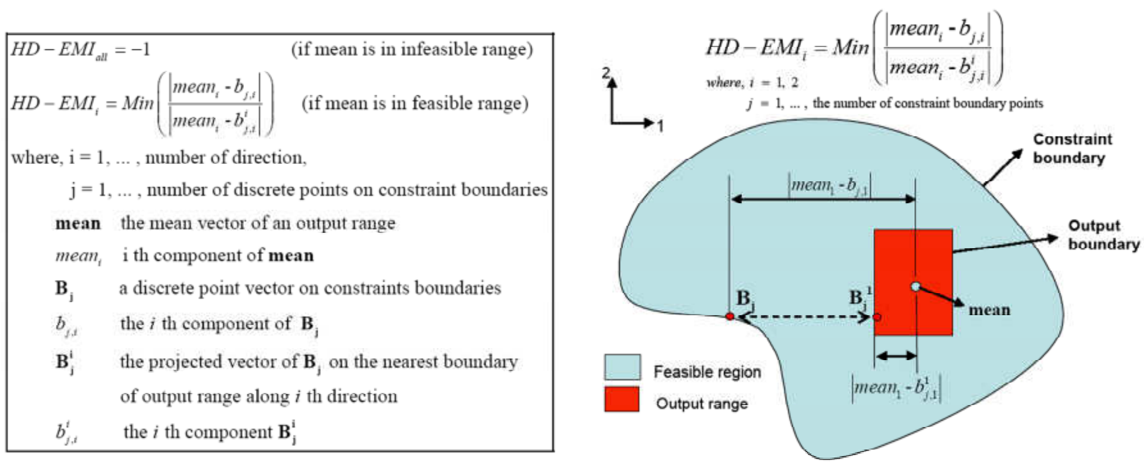


Figure 3.5 – Calculation of HD-EMI (Choi 2005)

Practically, HD-EMI is a measure of system's robustness. HD-EMI is a measure of the distance of a design point from design space boundary divided by variation in system performance. From Figure 3.5, note that as the HD-EMI increases, the output is more likely to be satisfactory, meaning that the output range moves farther from the constraint boundary, and system performance variation decreases (Choi 2005). Solutions that are far from the boundaries of the feasible design space will remain within the feasible design space in the presence of slight variation (i.e., these solutions are insensitive to, or robust to variation). Therefore, when determining a robust design solution, a designer is interested in selecting ranges with high values of HD-EMI. This approach to inductively determining ranges of design solutions is implemented in the work of Choi (Choi 2005). However, this approach is modified slightly in the example problems in this thesis. In this thesis, a single robust solution (rather than a range of solutions) is obtained by choosing the design point that best meets the design goals of maximizing product performance and maximizing system robustness (that is, maximizing HD-EMI). Instead of selecting a range of possible solutions that meet some minimum value of robustness (HD-EMI), in this thesis, the solution with the highest value of robustness (HD-EMI) while best meeting performance goals, is chosen as the inductive design solution.

For the multilevel example problems in this thesis, a single design solution rather than a range of design solutions is determined in order to more accurately gage the effectiveness of the multilevel design template. By solving for a single design solution, one is able to analyze the domain-specific performance validity of the multilevel design template. Additionally, for the BRP design problem, a single design solution is provided in order to meet customer requirements. Simple changes in the solution search algorithm implemented in the multilevel design template would result in a range of all possible solutions that meet a specified minimum level of robustness. The advantages of determining a range of possible design solutions and a single robust design solution are as follows. The main advantage of returning a range of all possible solutions is that the designer is able to select the best design solution from the range of satisficing design solutions based on intuition and designer expertise, giving the designer considerable flexibility. Advantages of returning a single design solution are that the best solution is presented when the complexity of the design problem is beyond designer intuition, and the domain-specific performance validity is more easily assessed when a single design solution is presented.

3.2 MULTILEVEL DESIGN TEMPLATE

Section 3.2 contains information relating to the creation and personalization of design templates as part of a multilevel design process. To begin, a discussion of extending the value of existing design methods with the development of design templates is given. Next, a generic formulation of a multilevel design template based on IDEM is presented. Then, the underlying assumptions and overall usefulness of the developed multilevel design template are given. Section 3.2 includes a discussion on the extent to which a generic template can be applied to a variety of design problems. The section concludes with the particularization of the generic multilevel design template for implementation in the example problems presented in Chapter 4 (design of a cantilever beam and its associated material) and Chapter 5 (design of a BRP).

3.2.1 From IDEM to a Multilevel Design Template

One of the key contributions of the work presented in this thesis is an illustration of how an existing design method can be transformed into a reusable, archival, modular design template. Recall from the glossary of key terms that a design template is a design process having a preset format, used as a starting point for a particular design application so that the format does not have to be recreated each time it is used (Panchal, et al. 2005). At the heart of every design method is a series of steps that, when followed properly, produce a specified design outcome. The steps in a design method can be expressed in a design template without loosing any of the inherent value in the design method. While many design methods are archival and reusable, they often are not modular or flexible. The advantage of creating design templates from design methods is that the valuable information in a design method can be transformed to a user-friendly, reusable, modular state, while method procedure is preserved.

Modularity is one of the key benefits of design templates. In complex design problems it is often beneficial to combine aspects from various design methods. However, modularity is not typically a characteristic that is designed into design processes. This lack of modularity makes it difficult to build a design method based on portions of existing design methods. By adapting a design method to a design template, the design method can be expanded or condensed based on the needs of a particular design problem. An example of design modularity is illustrated in Figure 3.6 with Legos®. Legos are generic modular building blocks that can produce a variety of designs.

Elements of a design template (also called sub-templates)

Overall design template

Figure 3.6 – Building blocks of a design template[3.1]

In Figure 3.6, an overall design template is illustrated as a ship built out of many Legos®. The overall design template is modular—it can connect with other Lego® creations. Additionally, the components used to build the ship are also modular. The individual building bricks are analogous to sub-templates that combine to describe an overall design process. This modularity within an overall design template and among other design templates highlights one of the key advantages of template-based engineering design.

[3.1] Images from www.lego.com

Another advantage of transitioning from a design method to a design template is that information produced at any stage in a design process can be stored in a design template for retrieval and use at a later stage in the design process. Design templates are particularly useful for multilevel design problems in which design information (such as goals, preferences, constraints, and bounds) is common to multiple design levels. By organizing information in a multilevel design process in the form of a design template, various sub-templates can be used to expresses the differences between levels, while another sub-template contains information that is used by all levels in a multilevel design process.

The previously stated procedure and advantages of template-based design are illustrated in the creation of the multilevel design template presented in Section 3.2.2. The multilevel design template is based on an existing design method, IDEM (Choi, et al. 2005). The main steps in IDEM are used to develop modules in the design template. For example, in Step 1, the designer is instructed to define relevant levels in a multilevel design process. This step is realized when a separate system response model is created for each level of a multilevel design process. Focusing on one step of IDEM at a time, a design template for multilevel robust design is adapted, and is presented in Section 3.2.2.

3.2.2 Formulation of Multilevel Design Template

In the following section, a general form of a multilevel design template is presented and discussed. The multilevel design template represents design templates at a high level of abstraction. That is, the general design template provides the framework for solving multilevel design problems. However, it is left to the designer to particularize the generic template into a form that is useful in providing design solutions to specific problems.

Multilevel Design Template

A word formulation of a multilevel design template is given in Figure 3.7. The construct of the word formulation is leveraged from the compromise Decision Support Problem (Mistree, et al. 1993a; Mistree, et al. 1993b). At this level of abstraction, the connection between the steps in the base method and the headings of the word formulation of a generic multilevel design template are easily observed.

Compromise Decision Support Problem	Multilevel Design Template
Given	**Given**
A feasible alternative	A feasible alternative
Assumptions	Assumptions
Parameters	Parameters
Goals	Goals
Find	**Define**
Design variables	Design levels
Deviation variables	Feasible design space
Satisfy	**Map**
Constraints	Discrete points to solution ranges (deductive)
Bounds	Solution ranges to robust solution (inductive)
Goals	**Find**
Minimize	Design variables
Weighted sum of deviation variables	Deviation variables
	Satisfy
	Constraints
	Bounds
	Goals
	Minimize
	Weighted sum of deviation variables

Figure 3.7 – Word formulation of generic multilevel design template compared to cDSP

The word formulation of the multilevel design template begins with information that is available to the designer at the beginning of the design process: goals, preferences, design variables, and design variable bounds. A “robustness goal” is incorporated in the multilevel design template in order to achieve robust solutions. For multilevel robustness examined in this thesis, the robustness goal is based on IDEM. The robustness metric (HD-EMI) defined in IDEM is maximized in order to maximize robustness of a multilevel design solution. Next, various levels in the design problem are defined along with the feasible design space at each level. The feasible design space is defined based on bounds placed on design variables. Next in the multilevel design template, deductive mapping functions are developed to describe the change in design variables and uncertainty models among the various levels of the design problem. By employing deductive mapping functions in an inductive manner, a robust solution is obtained such that design constraints and goals are met.

The word formulation of the multilevel design template is translated into a visual representation, illustrating information flow in the multilevel design template. The schematics of the design template are borrowed from an electrical breadboard which is a modular, reusable devise used to design electrical circuits. A generic design template at a single level is given in Figure 3.8.

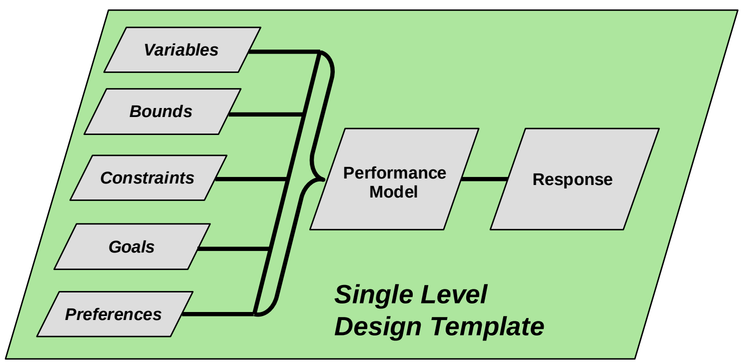


Figure 3.8 – Generic design template for a single level design process (Mistree, et al. 1993a; Mistree, et al. 1993b, Panchal, et al. 2004)

The single level design template in Figure 3.8 can be combined with other single level design templates to form a multilevel design template, as shown in Figure 3.9. (Single level design templates can also be joined in a horizontal manner). Mapping functions are developed to describe the flow of information among various levels. Moving from Level 1 to Level n represents an increase in model precision or complexity, in other words, changing from general models to specific models. As single level design templates are joined to form a multilevel design template, each single level design template becomes a sub-template of the overall multilevel design process. A sub-template is a design process building block linked with other building blocks to create a multilevel design template.

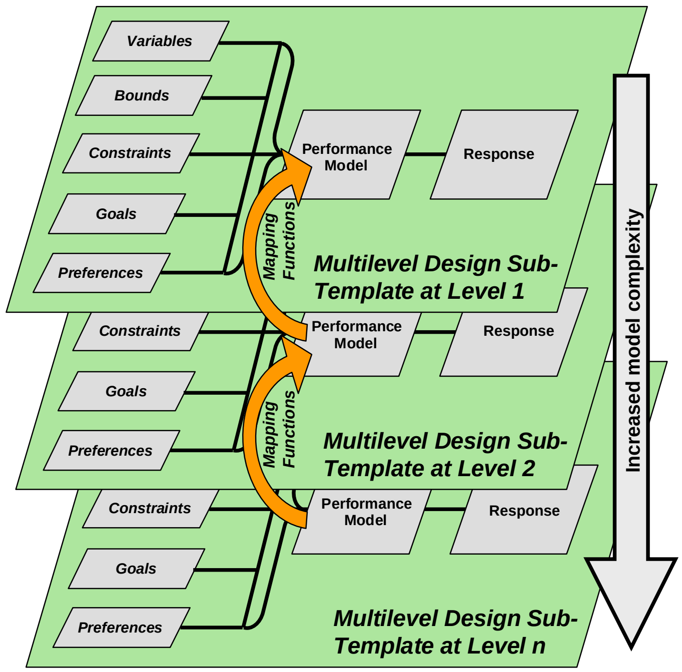


Figure 3.9 – Generic design templates for single level design combined in a multilevel design process

The multilevel design template presented in Figure 3.9 is divided into various sub-templates and simplified, shown in Figure 3.10. Information that is common to design at all levels is separated into the design parameter sub-template. The design parameter sub-template is used to store and access design information regarding design variables, design variable bounds, constraints, goals, and designer preferences. Analysis sub-templates, which are system prediction models developed by the designer, are distinguished at each design level. By partitioning the design process into various sub-templates, the modularity of the overall multilevel design template increases.

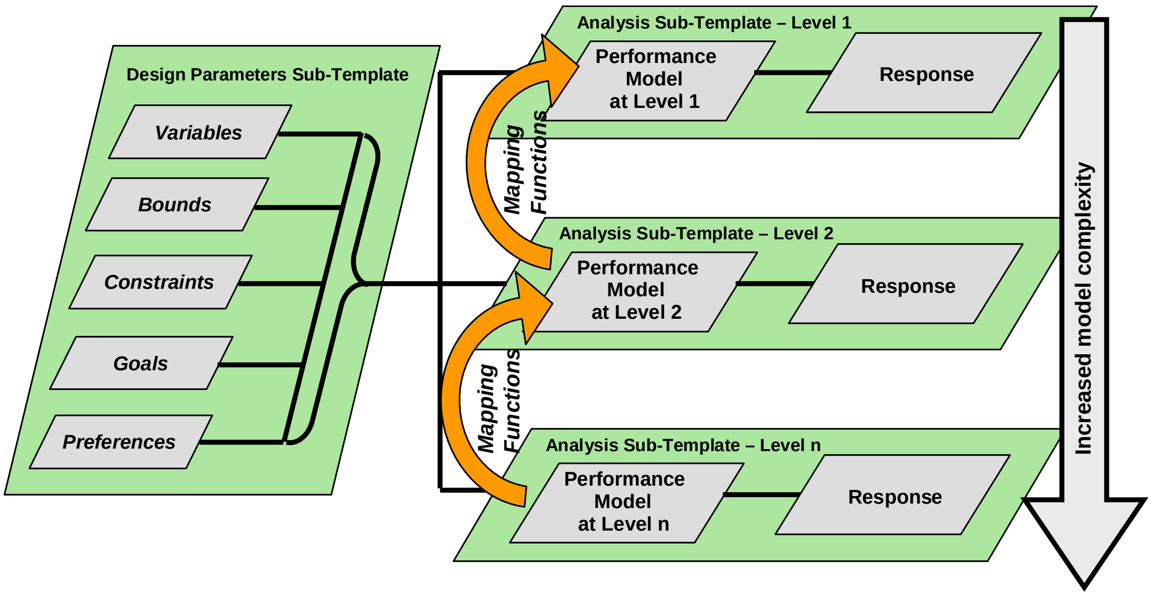


Figure 3.10 – Simplification of generic multilevel design process from previous figure

The development of mapping functions to link design information among various levels is one of the key steps in the multilevel design template. Mapping functions are mathematical expressions used to transfer design information among design levels. In complex multilevel design problems, mapping functions are used to limit design space at complex design levels by identifying likely regions containing satisficing design solutions. Initially, the structure of mapping functions are created in a deductive manner; that is, mapping functions describe design information from a more complex level to a less complex level using approximation or homogenization techniques. Then, using an inductive solution approach, the previously developed mapping functions are applied in an inductive manner. For example, in an inductive multilevel solution approach, a design solution is determined at the least complex design level, Level 1. Then, Level 1 design information is passed from Level 1 to Level 2 via a previously developed mapping function. An additional design constraint (based on the mapping function) is added at Level 2 such that the Level 2 design solution behaves similarly to the Level 1 design solution. In this way, the feasible design space at Level 2 is reduced allowing a satisficing design solution to be determined at reduced cost. In design problems with few design variables limiting the design space is not beneficial since an exhaustive search can be applied to the design space to determine the “best” design solution. However, increasing in design complexity to 10, 100, or 1000 design variables, fully exploring the design space is no longer a reasonable option. As a multilevel design problem increases in complexity, design information from all previous design levels is used to identify a reduced design space likely containing a satisficing design solution.

The multilevel design template presented in the previous section is a generic tool that can be particularized and implemented in specific design problems. In the following section assumptions and the overall usefulness of the developed template are explained. In Section 3.2.3, the generic multilevel design template is particularized for two example problems included in this thesis.

Multilevel Design Template – Assumptions and Usefulness

The developed multilevel design template has several key assumptions. First, in this thesis, levels in a multilevel design problem are divided according to model precision or model complexity. This assumption is a more general way of measuring design levels compared to what is observed in materials design. In the materials design community, a multilevel (also called multiscale) design problem involves design at various length scales. By distinguishing levels as measures of model complexity the essence of transitioning from general to specific in a multilevel design problem is captured.

Another assumption is that the method for incorporating robustness in a multilevel design problem in which the multilevel design template is utilized is left to the discretion of the designer. In IDEM (and in the multilevel design template), robustness is achieved by setting a goal to maximize a robustness metric, HD-EMI. HD-EMI is a measure of the distance from the current design point to the design bound divided by variation in system response (See Figure 3.5). In the example problems, the HD-EMI robustness metric is maximized to achieve robust solutions. However, due to the modular nature of the multilevel design template, additional methods for achieving robust designs (such as minimizing system response variation) could be applied without significantly affecting the structure of the design process.

Key assumptions relating to mapping functions in multilevel design follow. Mapping functions for solving a multilevel design problem are determined by the designer. It is assumed that there are many possible mapping functions for a particular design problem. It is the role of the designer to determine mapping functions to best transfer relevant design information while appropriately limiting the design space at more complex design levels. The mapping functions in multilevel template-based design have a significant impact on the overall design solution. Therefore, determining appropriate mapping functions represents a crucial decision point early in a multilevel design process. A formal procedure for "mapping function design" is not provided in this thesis. However, for future work in multilevel template-based design it is beneficial to investigate the influence of mapping functions in multilevel design solutions and to formulate a procedure for developing appropriate mapping functions for particular multilevel design problems.

The key advantages of the developed multilevel design template are modularity, flexibility, and ease-of-use. The developed template has two forms of modularity. The design template can be joined with other design templates to solve complex multilevel design problem, and the individual components of the template can be altered without negatively affecting the remainder of the template. Figure 3.11 illustrates how multilevel design templates can be combined in order to solve complex engineering problems involving multiple multilevel systems.

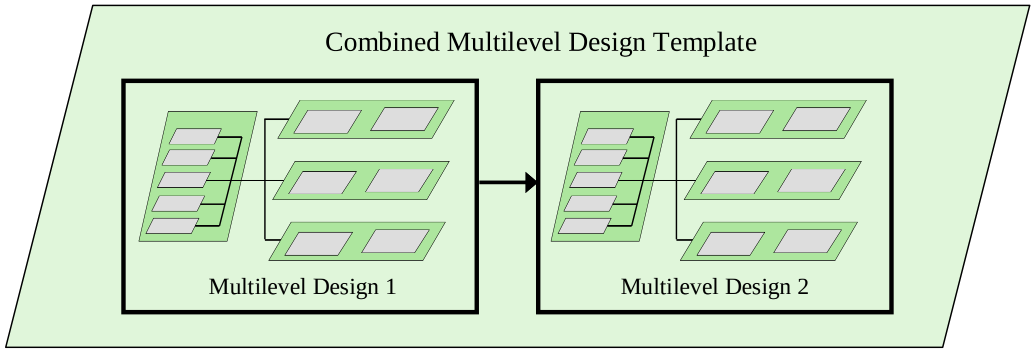


Figure 3.11 – Combined multilevel design template

The second aspect of modularity of the multilevel design template is that individual components used to develop the template provide modularity within the template itself. For example, the design parameters sub-template can be adjusted to reflect the needs of a specific design problem. Also, the analysis template can be duplicated in order to represent a multilevel design problem with considering many levels. The modular nature of the template relate to its flexibility in adjusting to the needs of individual design problems. The modularity, flexibility, and natural flow of information in the design template contribute to its ease-of-use.

3.2.3 Multilevel Design Template Particularized for Example Problems

In order to build confidence in the verification and validation of the multilevel design template, the template is applied to example problems and the value of design solutions is assessed. In the following section, the multilevel design template is particularized for two example problems. The example problems include the design of a cantilever beam and the design of a blast resistant panel. Additional information relating to design procedure for each of these example problems can be found in Chapter 4 (cantilever beam design) and Chapter 5 (BRP design). The successful application of the design template to these example problems builds confidence in its domain-specific structural validity and domain-specific performance validity (see Validation Square, Section 2.5).

Recall one of the main advantages of the multilevel design template is that it provides a procedure for limiting design space at complex design levels by searching design regions which are likely to contain satisficing design solutions. Complex design space reduction is achieved by mapping functions which translate design information from less complex design levels using averaging and homogenization techniques. This key advantage is examined in the application of the multilevel design template to the example problems. Containing at most 13 design variables, the example problems in this thesis lack the complexity of many product and material design problems encountered in the engineering design community. The example problems in this thesis can be directly solved at their most complex design levels and do not require the use of the multilevel design template to reduce design space at complex levels. However, the example problems are chosen to illustrate concepts relating to the key advantages of the multilevel design template. It is assumed that by applying the multilevel design template to more complex designs, the advantages of this design approach will be more apparent. A more complex design problem was not selected for examination in this thesis because the expertise required to model a complex design problem is beyond the scope of this research.

Multilevel Robust Design Template for Cantilever Beam Design

The multilevel design template is particularized for the design of a cantilever beam and its associated material. A cantilever beam with a square cross section is under constant loading at the free end. The design goals include minimizing the mass of the beam while maximizing beam robustness to uncertainty in material properties. In order to more closely achieve design goals, the material properties of the beam are allowed to vary along the length of the beam. This design problem, involving the concurrent design of product and material, represents a multilevel design problem.

To begin, it is determined that the various levels in the cantilever beam example problem are distinguished based on model complexity. At Level 1 a beam with constant material properties is observed. As the level increases, the material properties of the beam vary along the length of the beam according to discrete segments. At the greatest level of model complexity, the material properties of the beam are described as continuous functions along the length of the beam. The next step is to determine the number of design levels to consider in the design problem. Three levels that sufficiently describe the behavior of the beam are selected. The three levels are: a beam with constant material properties, a beam divided into 10 discrete segments each with independent material properties, and a beam in which the material properties are described as continuous functions. This multilevel design problem could be divided into many more levels of complexity (e.g., the microstructure and nanostructure of the beam's material could be designed). However, this example problem is intended to illustrate the concepts of template-based multilevel robust design. Next, deductive mapping functions are developed. These mapping functions describe material property mappings among the various design levels. Finally, a robust solution is determined based on inductive solution-finding techniques. Recall the design procedure of IDEM given in Figure 3.3 and Figure 3.4. The generic multilevel design approach is particularized for the cantilever beam design problem and is shown in Figure 3.12.

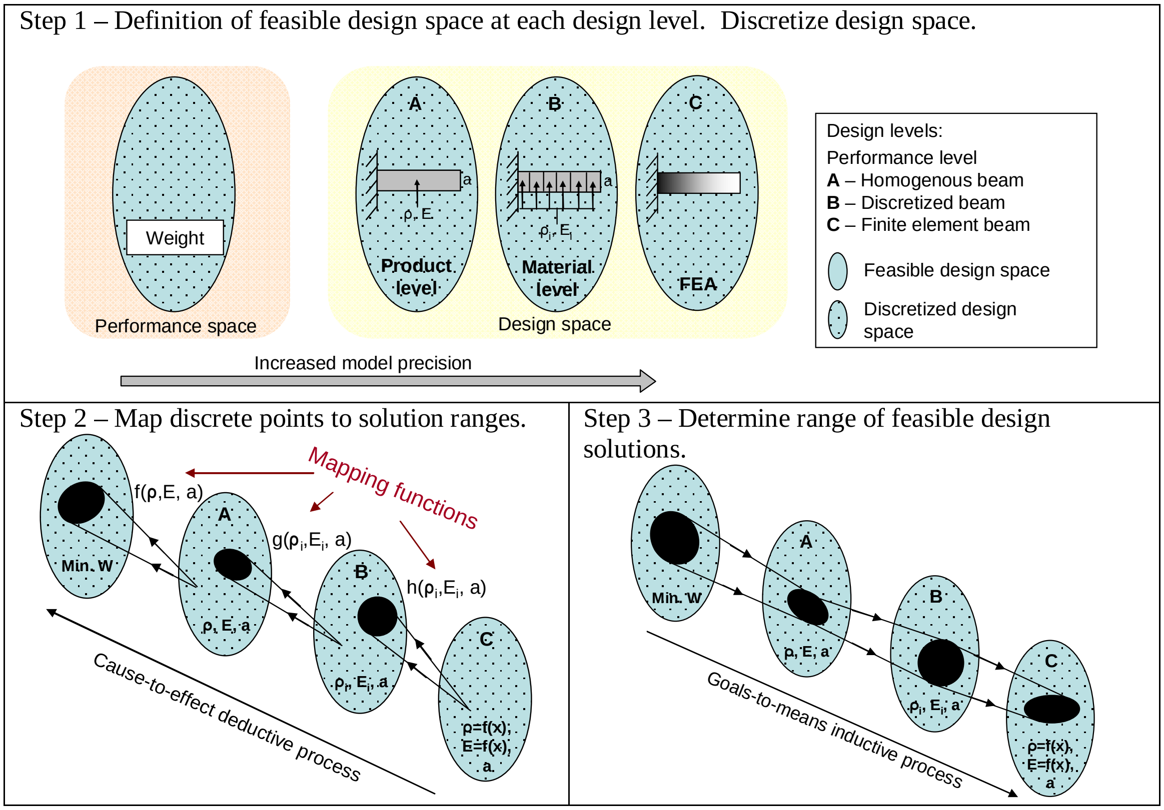


Figure 3.12 – Diagram of the inductive design exploration concept for cantilever beam design problem

The multilevel design template applied to the cantilever beam example problem is shown in Figure 3.13. Once the generic template is applied to a specific multilevel design problem, computational tools are introduced. The design template presented in Figure 3.13 is adapted to computer executable modules. By completing example design problems, the advantages of a template-based approach to multilevel robust design are observed, and confidence is built in the validation of the developed multilevel design template. Details regarding the implementation of the template in Figure 3.13 are in Chapter 4.

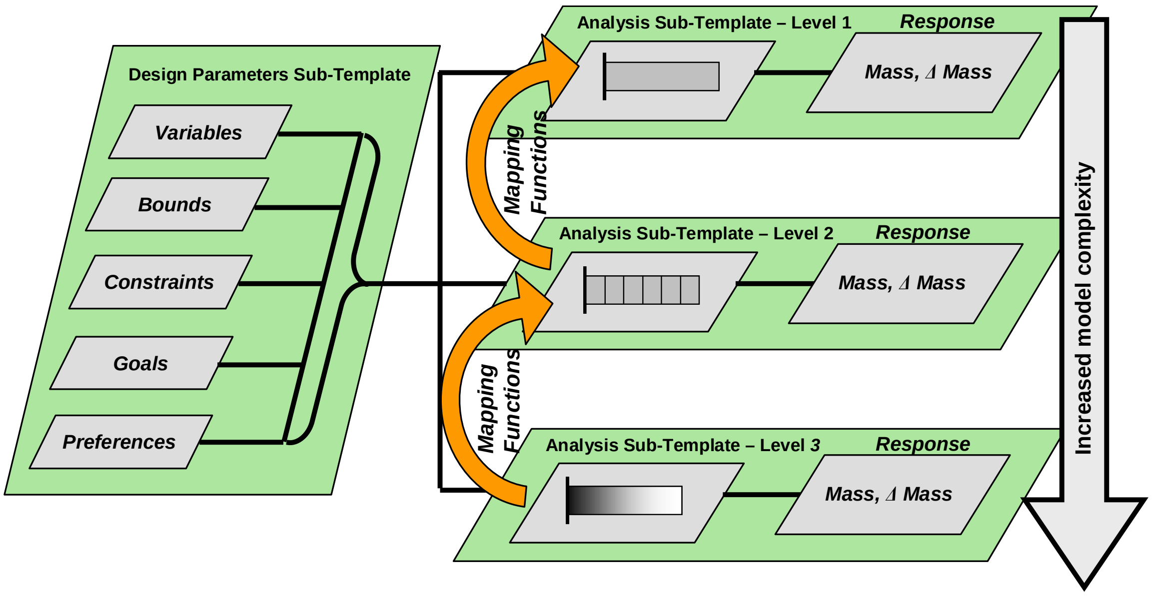


Figure 3.13 – Multilevel robust design template for cantilever beam example problem

Multilevel Robust Design Template for Blast Resistant Panel Design

The generic multilevel design template is particularized for the design of a BRP. A BRP is a sandwich structure consisting of a solid front and back face sheet surrounding a honeycomb core. Under impulse loading, a BRP experiences less deflection that similarly loaded panels of equal mass. The design goals include minimizing the deflection of the BRP, minimizing mass of the BRP, and maximizing BRP robustness with respect to uncertainty in loading conditions and material properties.

The various levels in the BRP example problem are distinguished based on model complexity. At Level 1, a BRP consisting of a solid panel is analyzed. As level increases, more details of BRP design are modeled and analyzed. At the greatest level of model complexity considered in this thesis, the BRP is modeled as a sandwich structure with two solid panels surrounding a honeycomb core. Additional levels of complexity are open for consideration in this design problem. One of the main benefits of applying a multilevel design template to this example problem is that the template can be updated when it becomes desirable to consider additional design features at more complex levels, such as a fill material in the core cells, or the design of associated material. Next, the number of design levels to consider in the design problem is determined. Three levels that sufficiently describe the behavior of the BRP are selected. The three levels are: a BRP modeled as a single panel (Level 1), a BRP modeled as three solid panels (Level 2), and a BRP modeled as two solid panels surrounding a honeycomb core (Level 3). This multilevel design problem could be divided into many more levels of complexity (e.g., the microstructure and nanostructure of the BRP material could be designed). However, such computationally expensive modifications are beyond the scope of this thesis. In Step 2, deductive mapping functions are developed. These mapping functions describe material property and uncertainty mappings among the various design levels. Finally, a robust solution is determined based on inductive solution-finding techniques. Figure 3.14 displays a diagram of the base method as applied to the BRP example problem.

The multilevel design template applied to the BRP beam example problem is shown in Figure 3.15. Once the generic template is applied to a specific multilevel design problem, computational tools are introduced. Similarly to the cantilever beam example, the design template presented in Figure 3.15 is adapted to computer executable modules. Details regarding the implementation of the template in Figure 3.15 are in Chapter 5 on BRP robust design.

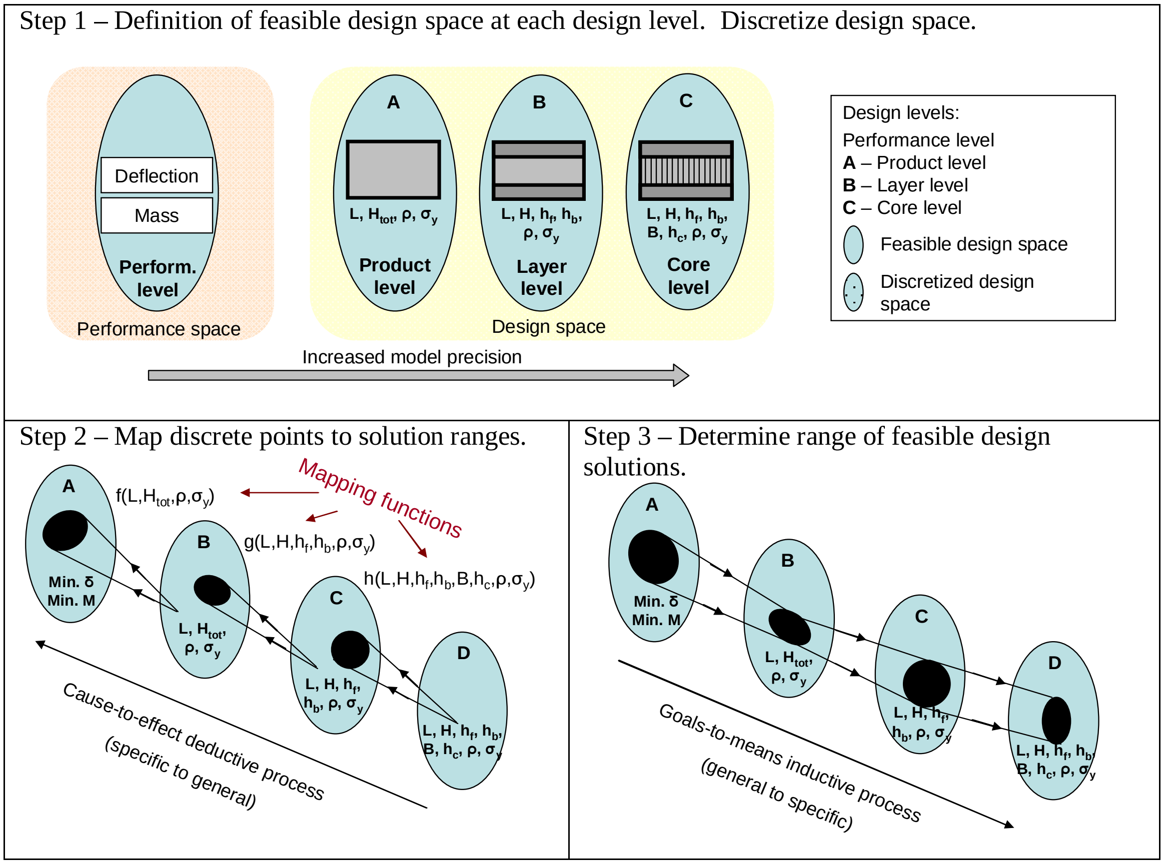


Figure 3.14 – Diagram of the inductive design exploration concept for BRP design problem

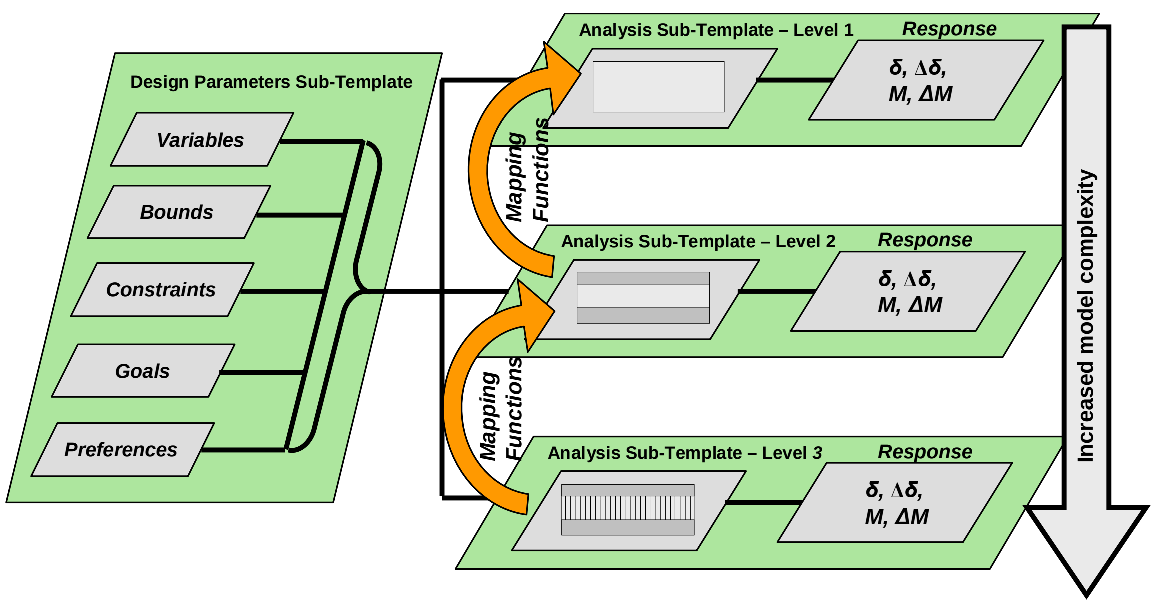


Figure 3.15 – Multilevel robust design template for BRP example problem

3.3 VERIFICATION AND VALIDATION OF TEMPLATE-BASED APPROACH TO MULTILEVEL DESIGN

In the following section, value is added to the verification and validation of the developed multilevel design template. To begin, the domain-independent performance validity of the multilevel design template is examined. Domain-independent performance validity relates to the internal consistency of the proposed template. Next, the domain-independent structural validity of the developed template is examined. Domain-independent performance validity is used to describe the likelihood that the template could be successfully applied to other design problems (outside of the examined fields) with positive design results. The following section on method validation provides only two pieces to the validation puzzle. At the end of Chapters 3 – 5, a discussion on method validation as it relates to the current chapter is given. In Chapter 6, the overall verification and validation of the developed template is discussed by examining verification and validation evidence presented at the end of each chapter.

3.3.1 Domain-Independent Structural Validity

The domain-independent structural validity of a design method relates to its internal consistency. The multilevel design template developed in Chapter 3 is based on an existing multilevel robust design method, IDEM. Thus, it follows that if the base method is internally consistent, than the multilevel design template is also internally consistent. The internal consistency of the base method is examined in detail in the Ph.D. dissertation of Hae-Jin Choi (Choi 2005 [Chapter 4]). In Choi's work, the domain-independent performance validity of the base method is tested by completing a conceptual example problem—the design of a cantilever beam and its associated material. Based on the effective application of the base method to an example problem, Choi asserts that IDEM is internally consistent. Since the multilevel design template presented in this thesis is based on IDEM, it can be concluded that it is also internally consistent.

An additional test for assessing the domain-independent structural validity of the multilevel design template is in analyzing the information flow through the template to ensure that adequate input information is provided to each step, and adequate output information is provided for subsequent steps. In Figure 3.16, an information flow chart for the multilevel design template is presented. An explanation of information flow in the multilevel design template follows. As shown in Figure 3.16, all design information originates from the designer. Additionally, there are several decision nodes in a multilevel design process requiring designer expertise denoted in Figure 3.16 with miniature designer icons. These critical decision nodes include formulating design parameters, developing performance models, creating mapping functions, and determining a multilevel solution approach.

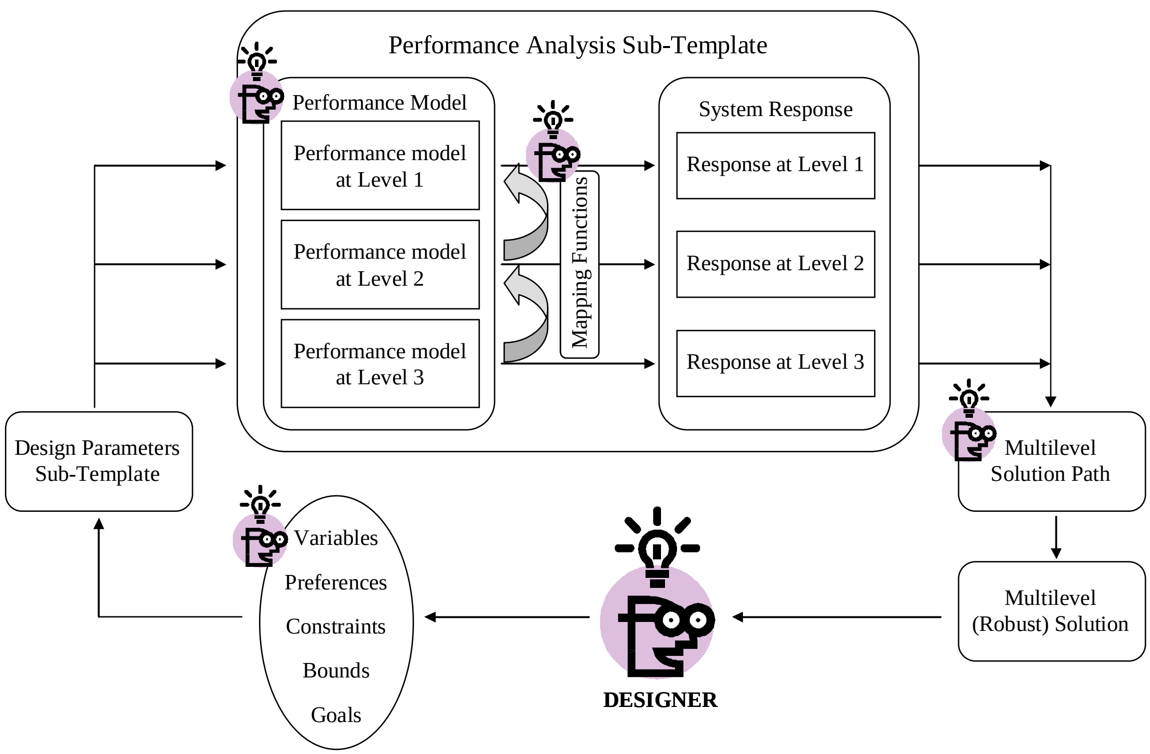


Figure 3.16 – Information flow chart for multilevel design template

As shown in Figure 3.16, at the beginning of a template-based approach to multilevel design, a designer specifies basic design information such as variables, preferences, constraints, bounds, and goals. This information is stored in a design parameters sub-template. The information from the design parameters sub-template is serves as input information to multilevel performance models. Design information passes from the design parameters sub-template to performance models by a system of storing and recalling information in a computational framework.

The next step in the multilevel design template is to define various levels of model complexity and develop system performance models at each level, steps provided by the designer. The designer also creates mapping functions which illustrate the mathematical relationship between various multilevel performance models and work to limit the unmanageable large design space at complex design levels. Developing multilevel performance models and mapping functions are the two most important steps of a multilevel design process because they have the most influence on the design solution. Input design information for the multilevel performance models comes from the design parameters sub-template. Output information of the performance analysis sub-template is a multilevel system response. The multilevel system response is then analyzed using a multilevel solution path strategy, provided by the designer. In this thesis, an inductive solution path strategy is used to analyze the multilevel response data to determine a multilevel design solution. The output information from the multilevel solution path is a multilevel design solution. If robustness goal is defined at the beginning of the design process in the design parameters sub-template, then the multilevel design solution will be robust to design uncertainty. To close the information loop of the multilevel design template, the multilevel design solution is directed to the designer for analysis. As shown in the multilevel design template information flow chart, each component of the template has adequate input information, provided from the designer or other template components, and each template component provides adequate output information for subsequent design process steps. This logical flow of information adds value to the internal consistency of the multilevel design template.

Additional confidence in the internal consistency of the developed template is achieved in the successful implementation of the template in two example problems. It can be assumed that applying the multilevel design template to two example problems and reaching successful outcomes indicates a logical flow of information within the design template. If the information flow in the multilevel design template is not logical, when the template is applied to example problems, it is assumed that such illogical or incomplete decision paths would be identified by the designer. A logical progression of thought and flow of information adds value to the domain-independent structural validity of the multilevel design template.

3.3.2 Domain-Independent Performance Validity

The domain-independent performance validity of a design method relates to the ability to apply the design method to a range of example problems, not previously tested, and achieve desirable results. The nature of a template-based approach to multilevel robust design adds value to its domain-independence performance validity. That is, design templates are designed to be sufficiently generic, modular, and flexible. Such qualities indicate that a design template can be applied to a range of design problems. The multilevel design template discussed in this thesis is presented at a high level of abstraction. That is, the design template is very general, and can be applied to most all multilevel design problems. However, in order to gain any real value from the design template, it must be particularized for a specific design problem. The key to the domain-independent structural validity of the multilevel design template is in its ability to be particularized to a variety of design problems. Based on the successful particularization of the template in the example problems presented in this thesis, it can be assumed that the design template can be applied to additional design problems, beyond the domain of examples investigated in this thesis.

The contribution of method validation presented in Chapter 3 is shown in Figure 3.17.

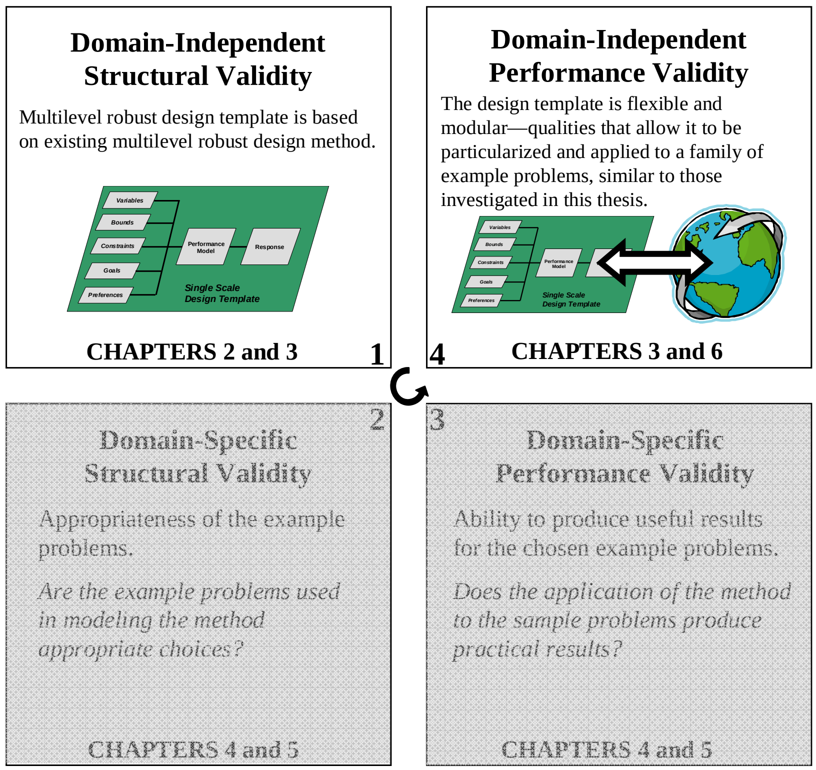


Figure 3.17 – Value added to verification and validation of design template in Chapter 3

3.4 SYNOPSIS OF CHAPTER 3

The multilevel design template presented in Chapter 3 provides the theoretical backbone for the reminder of the thesis. The multilevel design template is based on several key concepts from Chapter 1 (such as multilevel design, design templates, and method validation) and Chapter 2 (such as uncertainty in design, robust design, and the cDSP). Looking ahead in this thesis, two example problems are solved using the multilevel design template (Chapter 4 [cantilever beam], Chapter 5 [BRP]). In completing the example problems, the advantages of a template-based approach to multilevel design are illustrated. Additionally, successful completion of the example problems adds value to the verification and validation of the multilevel design template. In Chapter 6, research contributions from the development of the multilevel design template are presented and discussed. Information regarding the verification and validation of the multilevel design template are also presented in Chapter 6.

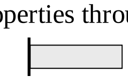
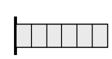
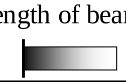
CHAPTER 4

CONCEPTUAL EXAMPLE – DESIGN OF A CANTILEVER BEAM

In Chapter 4, the concepts of template-based multilevel robust design are explored by completing the design of a cantilever beam and its associated material. A cantilever beam with non-negligible weight is loaded at the free end with a force of $F = 10$ N. The cantilever beam has a length $L = 2$ m and a square cross-section with characteristic length a. The maximum allowable deflection at the free end is $\delta_{\max} = 1$ cm. The design goals are to minimize the mass of the beam and to produce a design solution that is robust to uncertainty characterized by variation in material properties: density (ρ), elastic modulus (E), and yield strength (σ_y). Design variables include the cross-section area (a^2) and the material properties (ρ, E, σ_y) of the beam.

Materials design is achieved by alloying two existing materials (steel and aluminum) to produce a material with desired properties, and by functionally grading material properties along the length of the beam (from $x = 0$ to $x = L$). The desired material properties are achieved by controlling the volume fraction of steel or aluminum present in the designed material. Using the developed multilevel design template, the design problem is divided into various levels of increasing complexity, including the design of a solid beam with constant material properties, a discretized beam with independent material properties in each segment, and functionally graded beam with material properties that vary continuously along the length of the beam. Functions that map the properties and performance of the beam across levels are developed. Uncertainty propagation throughout the various levels is also modeled. Using a template-based approach and an inductive solution path, a multilevel cantilever beam that is robust to variation in material properties is designed. The completion of this example problem adds value to the verification and validation of developed multilevel design template by building confidence in its domain-specific structural and performance validity. A summary of the information in Chapter 4 is given in Table 4.1 and Figure 4.1 illustrates how Chapter 4 is connected to other ideas in this thesis.

Table 4.1 – Summary of Chapter 4

Heading / Sub-Heading	**Information**
Problem Overview	
Introduction to Example Problem	Cantilever beam example problem is introduced including: - Design requirements - Design goals
Design Approach	Nature of cantilever beam problem is examined: - Multilevel nature of design problem - Use of design templates for achieving inductive design solution
Value in Completing Example Problem	Address value based on: - Research questions presented in Chapter 1 - Verification and validation of a template-based approach to multilevel robust design
Design Process and Solution	
Particularization of Multilevel design template	Multilevel design template is organized under the headings: - Given – feasible alternative, assumptions, parameters, goals - Define – design levels, feasible design space, performance metrics - Map – discrete points to solution ranges (deductive), solution ranges to robust solution (inductive) - Find – design variables, deviation variables - Satisfy – constraints, bounds, goals - Minimize – weighted sum of deviation variables Multilevel design template is particularized for cantilever beam design problem
Design Process	Multilevel nature of cantilever beam design problem: - Level 1 – homogeneous material properties throughout beam  - Level 2 – discretized beam with independent material properties in each segment  - Level 3 – functionally graded beam with continuously changing material properties along length of beam 
Cantilever Beam Multilevel Inductive Design Solution	Method for inductive multilevel design is presented and design solutions are obtained and discussed
Verification and Validation	
Verification and Validation of Computational Design Tools	Verification and validation of the computational tools used in obtaining cantilever beam multilevel robust design solution
Verification and Validation of Multilevel design template	Value added to the verification and validation of a template-based approach to multilevel design based on completing beam design

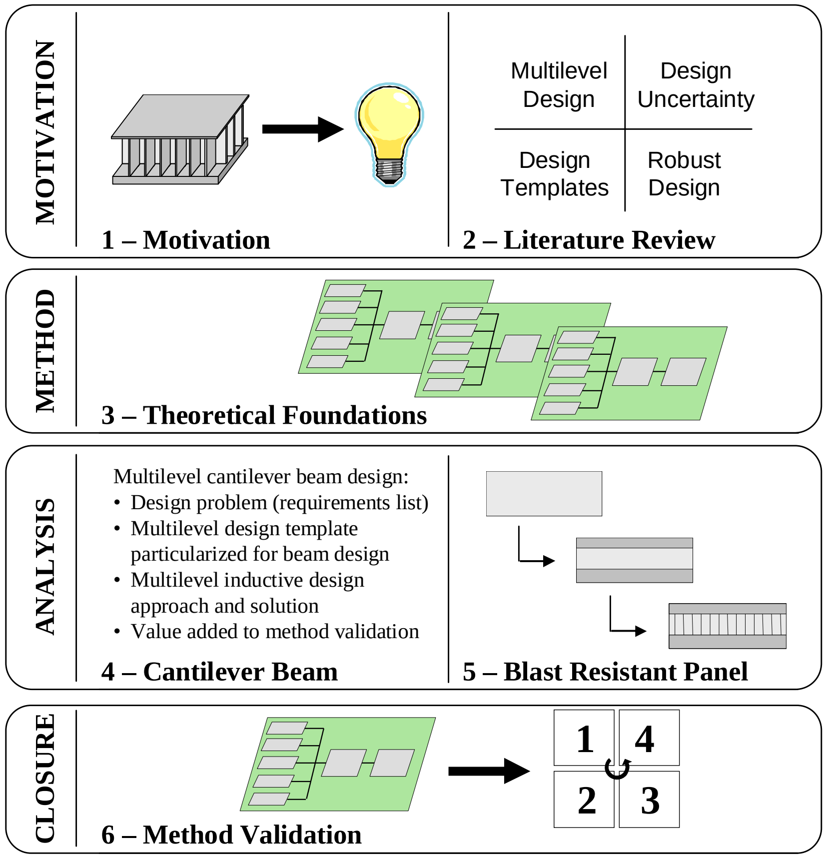


Figure 4.1 – Setting the context for Chapter 4

4.1 OVERVIEW OF CONCEPTUAL EXAMPLE – DESIGN OF A CANTILEVER BEAM

The example problem examined in Chapter 4 is used to demonstrate the implementation and advantages of a template-based approach in a multilevel robust design process. Using the generic multilevel design template presented in Chapter 3, the multilevel design template is particularized for the design of a cantilever beam and its material. Testing the presented template-based approach to multilevel design adds value to the verification and validation of the developed generic multilevel design approach from Chapter 3. In Section 4.2, an overview of the design problem is presented. A conference paper presented by Muchnick and coauthors at the *ASME IDETC/CIE* conference in 2006 details a preliminary cantilever beam design approach and solution (Muchnick, et al. 2006).

4.1.1 Introduction to Cantilever Beam Example Problem

Consider a solid cantilever beam with a square cross-section under constant loading. The base of the beam is clamped to a solid vertical surface, and the free end of the beam is allowed to translate and rotate. The beam is subject to a force of $F = 10$ N located at the free end. The weight of the beam is modeled as a constant distributed load along the length of the beam. The maximum allowable displacement at the free end is $\delta_{max} = 0.01$ m. The length of the beam is $L = 2$ m. In this design problem, the material properties of the beam, as well as the cross-section area of the beam are design variables. The design goals are to minimize the mass (thus, the weight) of the beam while designing a beam that is robust to variation in material properties. An illustration of the loading conditions and geometry of the beam are presented in Figure 4.2.

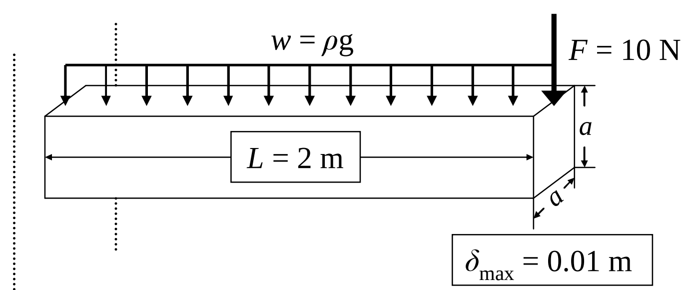


Figure 4.2 – Dimensions and loading conditions of cantilever beam

Incorporating materials design in the cantilever beam design problem gives an opportunity for increased design complexity due to a potentially large number of design variables. Because of this potential for design complexity, the cantilever beam design problem is divided into various levels of model complexity. In order to reach a final robust design solution, the various levels of design complexity are combined and analyzed in an inductive multilevel robust design approach. A generic approach for multilevel design is presented in Chapter 3. In order to apply this approach to the cantilever beam design problem, the multilevel design template is particularized for this design problem. This is discussed in more detail in Section 4.1.2.

Before the multilevel design solution is obtained, a method for analyzing beam performance at a single level is selected. A compromise Decision Support Problem (cDSP) is formulated at each design level in order to determine a robust design solution at a single level of model complexity. The cDSP is discussed in detail in Chapter 2. A cDSP for the overall cantilever beam design problem is given in Figure 4.3. A more mathematically rigorous cDSP is presented for each level of model complexity in Section 4.2.2. The design requirements and goals for the overall cantilever beam example problem are given in the cDSP in Figure 4.3 and discussed in more detail in the following paragraphs.

Table 4.2 – cDSP for multilevel cantilever beam design

Word Formulation	**Mathematical Formulation**
Given	*Given*
Cantilever beam with a square cross-section loaded at the free end with beam weight modeled as a distributed load	Force at free end: F = 10 N Beam length: L = 2 m Beam cross-section: $A = a^2$
Material properties reflect a steel / aluminum alloy using rule of mixtures where *vf* is percent of aluminum present in alloy	$X_{alloy} = (vf)\,X_{Al} + (1-vf)\,X_{Steel}$
Material property uncertainty model	$\Delta vf = 0.1$
Cantilever beam performance models: deflection mass safety factor	 $\delta = f\left(F, L, a, \rho, E\right)$ $m = f\left(a, L, \rho\right)$ $S.F. = f\left(F, L, a, \rho, \sigma_y\right)$
Find	*Find*
Beam design variables Beam deviation variables	Dimensions of beam cross-section: (a) Material properties of beam: (vf) d_i^+, d_i^-

Table 4.2 (continued) – cDSP for multilevel cantilever beam design

Satisfy	*Satisfy*
Constraints Maximum allowable deflection Minimum allowable safety factor	*Constraints* $\delta + \Delta\delta \leq 1$ cm s.f. ≥ 1
Bounds volume fraction density yield strength elastic modulus cross-section dimension deviation variables	*Bounds* $0 \leq vf \pm \Delta vf \leq 1$ $2700\ \text{kg/m}^3 \leq \rho \leq 7850\ \text{kg/m}^3$ $105\ \text{MPa} \leq \sigma_y \leq 325\ \text{MPa}$ $69\ \text{GPa} \leq E \leq 205\ \text{GPa}$ $0.5\ \text{cm} \leq a \leq 15\ \text{cm}$ $d_i^+, d_i^- \geq 0$ $d_i^+ \cdot d_i^- = 0$
Goals Minimize mass (m) Maximize HD-EMI^m	*Goals* $d_1^- = 1 - G_1 / A_1(x)$ $W_1 = 0.5$ $d_2^- = 1 - A_2(x) / G_2$ $W_2 = 0.5$
Minimize	*Minimize*
Deviation from target	$Z = W_1 d_1^- + W_2 d_2^-$

Cantilever Beam Design Requirements

The design requirements of the cantilever beam design problem are given in Table 4.3. The design requirements detail the geometric, loading, performance, material, and analysis specifications of the designed cantilever beam.

Table 4.3 – Requirements list for cantilever beam design

Design of a Cantilever Beam and its Material		**Requirements list for the multilevel design of a cantilever beam and its associated material**	**Issued On: 10/1/2006**
Problem Statement: Design a cantilever beam and its associated material to minimize beam mass and maximize robustness to material property uncertainty. Design variables include material properties (ρ, E, σ_y) and cross-section area (a^2).			
#	**Demand/Wish**	**Requirements**	
Geometric requirements			
1	D	Length of beam: $L = 2$ m	
2	D	Cross-section area of beam is square: Area = a^2 m^2	
3	D	$0.005\ \text{m} \leq a \leq 0.15\ \text{m}$	
Loading Requirements and Boundary Conditions			
4	D	Force at free end: $F = 10$ N	
5	D	Weight of beam is modeled as a distributed load along length of beam	

Table 4.3 (continued) – Requirements list for cantilever beam design

6	D	Beam is clamped at base (no translation or rotation) with no imposed boundary conditions at the free end
Performance Requirements		
7	D	Maximum allowable deflection: δ_{max} = 0.01 m
8	D	Safety factor ≥ 1
9	W	Minimize beam mass (performance goal)
10	W	Maximize HD-EMI (robustness goal)
Material Requirements		
11	D	2700 kg/m^3 ≤ ρ ≤ 7850 kg/m^3
12	D	69 GPa ≤ E ≤ 205 GPa
13	D	105 MPa ≤ σ_y ≤ 325 MPa
14	W	Uncertainty interval ρ = 10% of material property range = 515 kg/m^3
15	W	Uncertainty interval E = 10% of material property range = 13.6 GPa
16	W	Uncertainty interval σ_y = 10% of material property range = 22 MPa
17	D	Material properties ± ½ uncertainty interval must be within material bounds
18	W	Vary material properties along length of the beam to more closely achieve design goals
Computational Requirements		
19	W	Beam performance (deflection, mass, variance of mass) must be calculated using performance equations and / or finite element analysis

The geometric requirements are based on design feasibility and manufacturability. Performance requirements are selected due to expected beam performance and beam failure criteria. Material requirements are based on the material properties of aluminum and steel (www.matweb.com). Uncertainty in material properties is determined by the designer as a reasonable amount of material property variation for an alloying process.

Cantilever Beam Design Goals

There are two goals associated with the cantilever beam design problem: minimize beam mass and maximize robustness metric HD-EMI (hyper-dimensional error margin index). Calculating beam mass involves simple deterministic equations which are given in Section 4.1.2. Recall from Chapters 2 and 3 that HD-EMI is a metric for measuring the robustness of a system that contains uncertainty in noise factors, control factors, and propagated process chain uncertainty. For the cantilever beam design problem, it is assumed that the material properties (control factors) are uncertain. Therefore, by maximizing HD-EMI, a cantilever beam design that is robust to variations in material composition and robust to uncertainty in the multilevel design process chain is achieved. More details in how the HD-EMI metric is applied to the design problem at each level is given in Section 4.2.

4.1.2 Multilevel Design Approach for Cantilever Beam Design

In Section 4.1.2, an overview of the multilevel approach used in solving the cantilever beam design problem is presented. First, the multilevel nature of the design problem is presented. Then, a summary of the application of the multilevel design template (developed in Chapter 3) in the cantilever beam example problem is given.

Cantilever Beam Design as a Multilevel Design Problem

At first, the cantilever beam design problem may seem rather simple, and not multilevel. After careful consideration, it becomes clear that by designing the geometry *and* the material of the beam, this example problem has the potential for increased design complexity. For example, the cantilever beam material could be designed at the continuum, meso, micro, or nano level. Also, material properties could be homogeneous or vary throughout the beam. When presented with such design complexity, it is left to the expertise of the designer to determine the levels of complexity that are considered in this design problem.

In this thesis, the multilevel nature of the cantilever beam problem is defined based on three levels of increasing design complexity. At multilevel model 1, the beam is modeled with homogeneous material properties. The design variables include the material properties of the beam (ρ, E, σ_y) and the characteristic cross-section dimension (a). At multilevel model 2, the material properties of the beam are allowed to vary along the length of the beam. The beam is divided into 10 discrete segments with independent material properties. The material properties in each segment are a function of volume fraction. Volume fraction is a measure of the percent of aluminum present in the steel-aluminum alloy used in beam materials design. A variety of material properties (ρ_i, E_i, σ_{yi}, for i = 1 to 10) can be defined using only 10 volume fraction design variables. The design variables in multilevel model 2 include the volume fraction of each segment ($vf1$ – $vf10$) and the characteristic cross-section dimension (a). At multilevel model 3, the beam is designed using continuously changing material properties. Similar to multilevel model 2, volume fraction calculations are used to calculate beam material properties. The continuously changing material properties of the beam are determined based on a series of cubic splines. The design variables include 21 control points evenly spaced along the length of the beam (used in calculating cubic splines) and the cross-section dimension (a). The mathematical details for this multilevel design problem are presented in detail in Section 4.2. A pictorial representation of the multilevel nature of the cantilever beam design problem is shown in Figure 4.3.

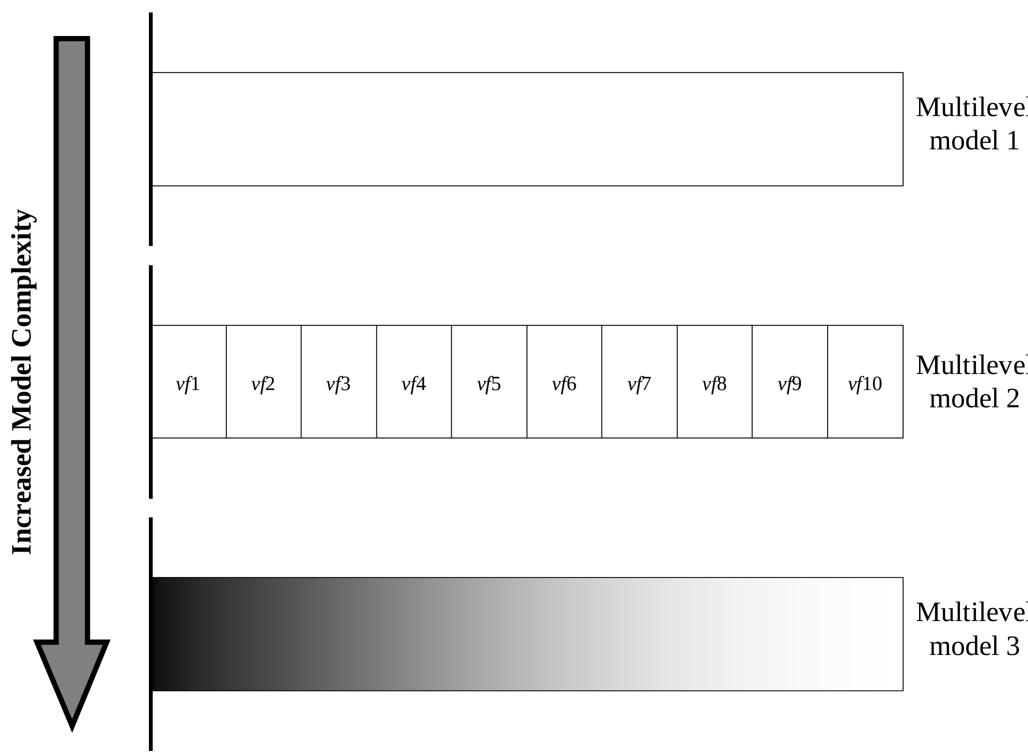


Figure 4.3 – Multilevel design approach for cantilever beam example

Application of Multilevel Design Template to Cantilever Beam Design

In Chapter 3 a multilevel design template is presented. This design template provides the framework for formulating and solving complex design problems. However, the multilevel design template has limited use until it is applied to a specific design problem. As seen in Section 4.1, the cantilever beam design problem is multilevel in nature. The multilevel design template is particularized and applied to the example problem in order to facilitate the systematic design of a cantilever beam and its associated material (see Figure 4.5). In Figure 4.4, the potential for augmenting the generic multilevel design template for cantilever beam design is shown.

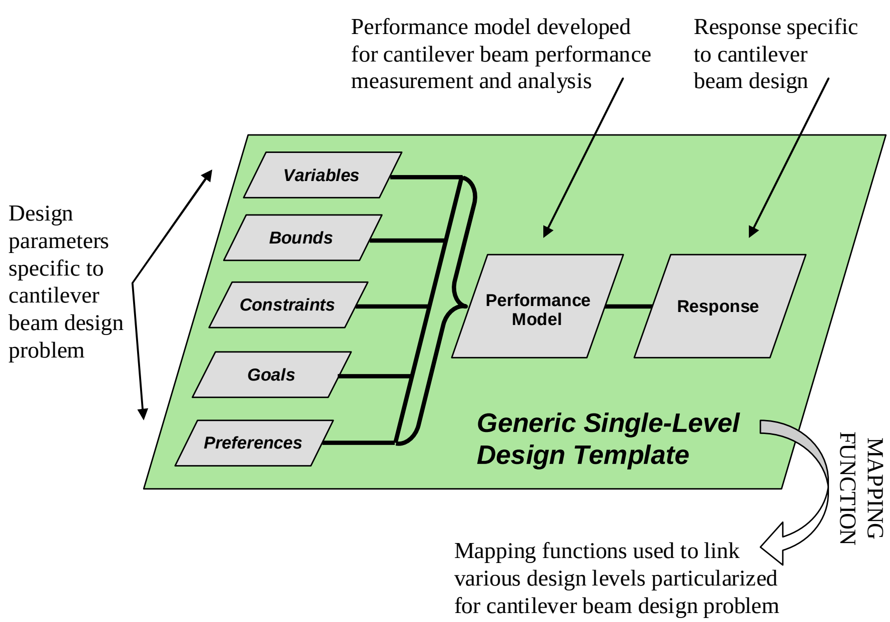


Figure 4.4 – Multilevel design approach for cantilever beam example

For the three levels of model complexity considered in the cantilever beam example, the generic single-level design template is particularized for the design parameters, performance model, and response at each level. Then, the single-level cantilever beam design templates are joined via mapping functions to create the framework for inductive multilevel cantilever beam design.

4.1.3 Value in Completing Cantilever Beam Example Problem

Addressing Research Questions

In the following section, the motivation for completing the example problem of the design of a cantilever beam and its material is discussed. The motivation for completing the example problem is divided in two topics: to demonstrate the multilevel robust design of multilevel systems using a design template approach (in response to Research Question #1, Section 1.3), and to provide an avenue for the validation of the generic multilevel design template presented in Chapter 3. A summary of the example problem and the related motivation is presented in Figure 4.5.

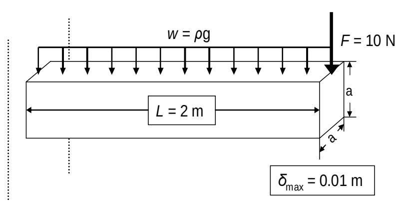
Example Problem	Motivation
Cantilever Beam Design Design of a cantilever beam and its associated material  	• Design goals: minimize beam weight; maximize beam robustness to uncertainty in material properties • Design constraints: maximum deflection $\delta_{max} \leq 1$ cm; safety factor ≥ 1 • Design variables: material properties of beam; beam cross-sectional area • Example problem used to illustrate the concepts of template-based multilevel robust design and to validate developed multilevel design template

Figure 4.5 – Overview and motivation for cantilever beam example problem

The cantilever beam example problem is selected because of its similarity to the topics discussed in the research questions. That is, the cantilever beam example problem is a clearly defined multilevel design problem for which a multilevel design template can be applied. The primary motivation in completing the cantilever beam example problem is to demonstrate the usefulness of a template-based approach in the robust design of multilevel systems (Research Question #1, Section 1.3). The application of the multilevel design template to the cantilever beam example problem, and evidence of useful results, add value to the verification and validation of the multilevel design template presented in Chapter 3.

Verification and Validation of Multilevel Design Template

Additionally, completing the cantilever beam example problem adds value to the verification and validation of the developed multilevel design template (discussed in more detail in Section 4.3 and in Chapter 6). The cantilever beam example problem contributes to the domain-specific structural validity (appropriateness of example problems) and the domain-specific performance validity (ability to produce useful results for the chosen example problems) of the multilevel design templates. Additionally, the cantilever beam example problem is intended to illustrate the key advantages of the multilevel design template. Recall that one of the key advantages of the multilevel design template is a procedure for limiting complex design space exploration to include only areas likely to contain satisficing design solutions. Although the cantilever beam example problem is relatively simple and can be solved directly without the use of the multilevel design template, cantilever beam design is included in this thesis to illustrate the benefits of the multilevel design template when applied to more complex design problems. This example problem is not intended to encompass a detailed, comprehensive multilevel robust design problem formulation and solution. However, lessons obtained in completing this conceptual example problem can be abstracted to facilitate deeper learning in template-based multilevel robust design. A more comprehensive example problem is presented and discussed in Chapter 5, in the design of a BRP.

4.2 CANTILEVER BEAM DESIGN PROCESS AND SOLUTION

In the following section, the design of a cantilever beam and its associated material are presented. First, the cantilever beam design process, as it relates to the developed multilevel design template, is discussed. Then, the details of the problem formulation, design process, and solutions are examined. The successful implementation of a template-based approach to the design of a cantilever beam and its material build confidence in the verification and validation of a template-based design approach to multilevel systems.

4.2.1 Multilevel Design Template Particularized for Cantilever Beam Design

In the following section, a generic template for multilevel robust design is particularized for the design of a cantilever beam and its material. Recall from Chapter 3 the generic design template for multilevel robust design represented in word form, organized under the headings *Given*, *Define*, *Map*, *Find*, *Satisfy*, *Minimize*, is restated in Figure 4.6. The generic multilevel design template is particularized for the cantilever beam example problem and is shown in Figure 4.7.

Given A feasible alternative Assumptions Parameters Goals **Define** Design levels Feasible design space **Map** Discrete points to solution ranges (deductive) Solution ranges to robust solution (inductive)	**Find** Design variables Deviation variables **Satisfy** Constraints Bounds Goals **Minimize** Weighted sum of deviation variables

Figure 4.6 – Generic word formulation of multilevel design template

The cantilever beam multilevel design template in Figure 4.7 begins with the collection and modeling of design parameters including: design variables, design variable bounds, design constraints, design goals, and design preferences. The design parameters are given in the problem statement or determined by an experienced engineer. Following the specification of design parameters, multilevel models of the multilevel design problem are developed. Multilevel models are used to predict the response or performance of the design at each level of model complexity.

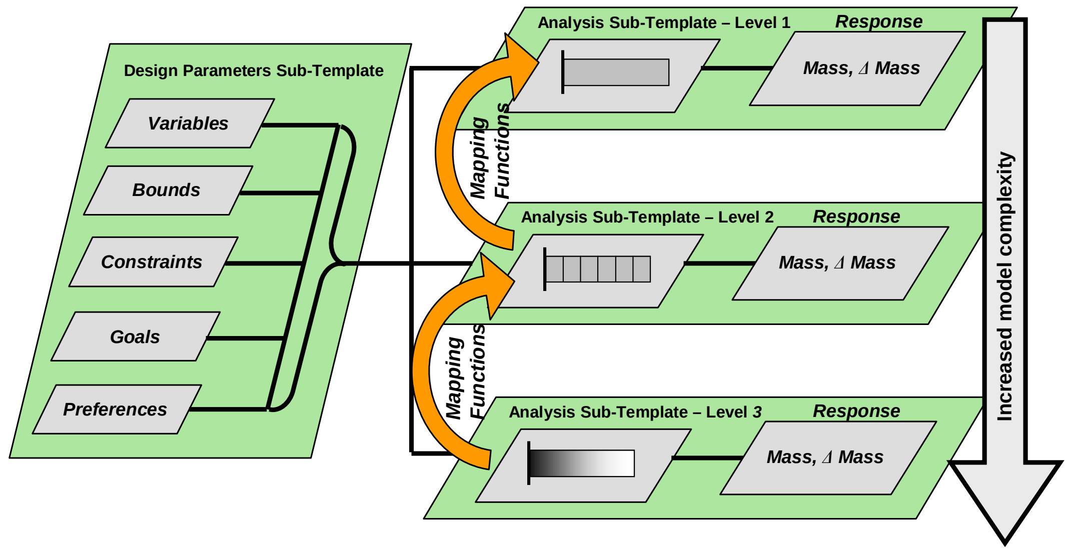


Figure 4.7 – Multilevel design template for cantilever beam product and materials design

For the cantilever beam design problem, three levels of model complexity are considered. The multilevel models defined for the cantilever beam example problem include a beam with homogeneous material properties, a discretized beam (10 segments of uniform length), and a beam with functionally graded material properties. The cantilever beam example problem could be described using more levels of complexity. However, for the purposes of this conceptual example problem, three levels of design complexity are sufficient to display the concepts presented in this thesis. Each multilevel model is linked with mapping functions that map material properties and uncertainty models at each level interface. At the end of the multilevel design template, the design performance is determined using multilevel models. For the cantilever beam example, the mass and variation of mass is measured at the *Design Performance* section of the multilevel design template.

4.2.2 Cantilever Beam Design Process

In the following section, the cantilever beam design process based on the developed multilevel design template is presented. In Section 4.2.2, the problem formulation and solution procedure is given. In Section 4.2.3 the inductive robust design solution is presented. Topics discussed in the following section are organized under the headings found in the multilevel design template: *Given*, *Define*, *Map*, *Find*, *Satisfy*, *Minimize*. By grouping the design information in this way, the natural flow of information in the multilevel cantilever design process is preserved.

Given

The following information provides the underlying assumptions of the cantilever beam design problem.

<u>A Feasible Alternative</u>

The feasible alternative that this design process is based on is a cantilever beam with a square cross-section. The beam is clamped at its base and is loaded at its free end. The weight of the beam is considered as non-negligible and is modeled as a distributed load along the length of the beam. Improvements to the feasible alternative are achieved my modifying the product geometry and material properties.

<u>Assumptions</u>

Assumptions used to model beam performance are as follows: The beam is assumed to be clamped at its base (no translation or rotation). The free end of the beam is allowed to translate and rotate in all directions; however, it is assumed that maximum beam deflection occurs in the same direction as the applied load (in the y-direction). Based on cantilever beam deflection equations, the maximum deflection is assumed to occur at the free end (at $x = L$). Materials design of the cantilever beam occurs at the continuum level.

Any change in material properties throughout the beam occurs along the length of the beam (in the *x*-direction).

Parameters

Parameters for cantilever beam design include system variables, constraints, and goals. Design parameters are summarized in Table 4.4, and discussed in more detail in the following section.

Table 4.4 – Cantilever beam design parameters

System Variables	Constraints	Goals
vf, *a* (Level 1)	$\delta_{max} \leq 1$ cm	minimize mass
vf1 – vf10, *a* (Level 2)	s.f. ≥ 1	maximize HD-EMI
cp1 – cp21, *a* (Level 3)		

Define

In the following section design levels and feasible design space are defined for the cantilever beam example.

Design Levels

Design levels represent the amount of design simplicity or complexity that is considered in reaching a design solution. The cantilever beam design problem is divided into three levels of model complexity. In the following section, a cDSP, performance modeling equations, and modeling techniques are presented for each design level. Recall that when making design decision at a single level, a cDSP is employed.

Level 1

In Level 1, the simplest level of cantilever beam analysis with two design variables, the material properties of the beam are considered to be constant throughout the beam. As stated previously, the material properties of the beam are achieved by alloying steel and aluminum. Therefore, material properties are expressed based on the volume fraction of aluminum present in the alloy. This relationship is shown in Equation 4.1, and can be used for all of the material properties required for material property calculations: E, ρ, and σ_y, where X_A and X_B are the material properties of alloy component A and alloy component B, respectively. By varying the volume fraction of the beam's material one is able to design a material that best meets design goals.

$$X_{alloy} = (vf)X_A + (1 - vf)X_B \tag{4.1}$$

A cDSP for Level 1 is given in Table 4.5. Recall the word formulation of the multilevel design template in Figure 4.6. Under the Define heading, three design levels are defined for the cantilever beam design problem. A cDSP is created at each design level (Table 4.4).

Table 4.5 – Cantilever beam Level 1 cDSP

Word Formulation	**Mathematical Formulation**
Given	*Given*
Cantilever beam with a square cross-section loaded at the free end with beam weight modeled as a distributed load	Force at free end: F = 10 N Beam length: L = 2 m Beam cross-section: $A = a^2$
Material properties reflect a steel / aluminum alloy using rule of mixtures where *vf* is percent of aluminum present in alloy	$X_{alloy} = (vf)X_{Al} + (1 - vf)X_{Steel}$
Constant material properties throughout beam	
Material property uncertainty model	$\Delta vf = 0.1$
Cantilever beam performance models: deflection mass safety factor	 $\delta = f(F, L, a, \rho, E)$ $m = f(a, L, \rho)$ $S.F. = f(F, L, a, \rho, \sigma_y)$
Find	*Find*
Beam design variables	Dimensions of beam cross-section: (a) Material properties of beam: (vf)
Beam deviation variables	d_i^+, d_i^-

Table 4.5 (continued) – Cantilever beam Level 1 cDSP

Satisfy	*Satisfy*
Constraints Maximum allowable deflection Minimum allowable safety factor	*Constraints* $\delta + \Delta\delta \leq 1$ cm s.f. ≥ 1
Bounds volume fraction density yield strength elastic modulus cross-section dimension deviation variables	*Bounds* $0 \leq vf \pm \Delta vf \leq 1$ 2700 kg/m$^3 \leq \rho \leq$ 7850 kg/m^3 105 MPa $\leq \sigma_y \leq$ 325 MPa 69 GPa $\leq E \leq$ 205 GPa 0.5 cm $\leq a \leq$ 15 cm $d_i^+, d_i^- \geq 0$ $d_i^+ \cdot d_i^- = 0$
Goals Minimize mass (m) Maximize HD-EMI^m	*Goals* $d_1^- = 1 - G_1 / A_1(x)$ $W_1 = 0.5$ $d_2^- = 1 - A_2(x) / G_2$ $W_2 = 0.5$
Minimize	*Minimize*
Deviation from target	$Z = W_1 d_1^- + W_2 d_2^-$

Determining a robust solution to the design of a cantilever beam at Level 1 is executed in Matlab. The cDSP is modeled as a multi-objective constrained optimization problem, making use of the Matlab built-in function `fmincon()` (MATALB 2004). The design variables include the characteristic cross-section dimension (a) and the volume fraction of beam material (vf). The performance of the beam is measured by calculating the deflection ($\delta_{\max}$), mass (m), variation of mass (Δm), safety factor (S.F.), and HD-EMI of beam performance. The equations used to calculate the performance of the beam are presented below (Gere 2001; Choi 2005).

$$\delta_{\max} = -\frac{\left(8FL^3 + 3\rho g a^2 L^4\right)}{2Ea^4} \tag{4.2}$$

$$m = a^2 L \rho \tag{4.3}$$

$$\Delta m = \left|\frac{\partial m}{\partial \rho}\right| \Delta\rho = a^2 L \Delta\rho \tag{4.4}$$

$$S.F. = \frac{\sigma_y}{\left(\frac{6L(F + \rho g / 2)}{a^3}\right)} \tag{4.5}$$

$$\text{HD} - \text{EMI}_{mass} = \min\left(\frac{\left|mean_{mass} - b_{j,mass}\right|}{\left|mean_{mass} - b_{j,mass}^{mass}\right|}\right) \tag{4.6}$$

Level 2

For Level 2 model complexity, the material properties of the cantilever beam are allowed to vary along the length of the beam, as shown in Figure 4.8.

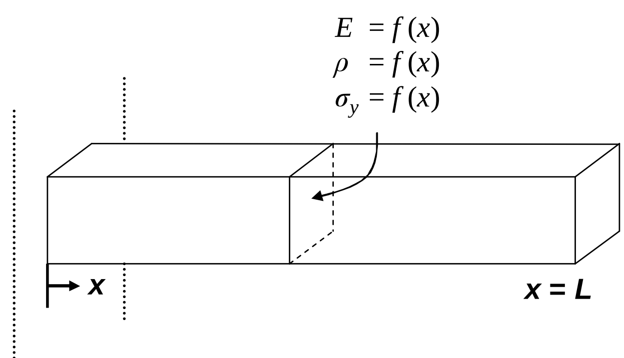


Figure 4.8 – Material properties as a function of *x*

Recall from Figure 4.3 that at Level 2, the cantilever beam is modeled with 10 discrete segments (each of length 0.2 m) such that the material properties of each segment are independent. A design solution that more accurately achieves design goals is realized by choosing a stronger/heavier material for the base of the beam, and a weaker/lighter material for the beam's free end. A cDSP for Level 2 is presented in Table 4.6.

Table 4.6 – Cantilever beam Level 2 cDSP

Word Formulation	**Mathematical Formulation**
Given	*Given*
Cantilever beam with a square cross-section loaded at the free end with beam weight modeled as a distributed load	Force at free end: F = 10 N Beam length: L = 2 m Beam cross-section: $A = a^2$
Material properties reflect a steel / aluminum alloy using rule of mixtures where *vf* is percent of aluminum present in alloy	$X_{alloy} = (vf)X_{Al} + (1 - vf)X_{Steel}$
Independent material properties in 10 segments of beam	
Material property uncertainty model	$\Delta vf = 0.1$
Cantilever beam performance models: deflection mass safety factor	 $\delta = f(F, L, a, \rho, E)$ $m = f(a, L, \rho)$ $S.F. = f(F, L, a, \rho, \sigma_y)$
Find	*Find*
Beam design variables	Dimensions of beam cross-section: (a) Material properties of beam: ($vf1, vf2, ..., vf10$)
Beam deviation variables	d_i^+, d_i^-
Satisfy	*Satisfy*
Constraints Maximum allowable deflection Minimum allowable safety factor Multilevel inductive constraint	*Constraints* $\delta + \Delta\delta \le 1$ cm s.f. ≥ 1 $\frac{\sum_{i=1}^{10} vf_{i,level2}}{10} = (\pm 5\%) vf_{level1}$
Bounds volume fraction density yield strength elastic modulus cross-section dimension deviation variables	*Bounds* $0 \le vf \pm \Delta vf \le 1$ 2700 kg/m$^3 \le \rho \le$ 7850 kg/m^3 105 MPa $\le \sigma_y \le$ 325 MPa 69 GPa $\le E \le$ 205 GPa 0.5 cm $\le a \le$ 15 cm $d_i^+, d_i^- \ge 0$ $d_i^+ \cdot d_i^- = 0$
Goals Minimize mass (m) Maximize HD-EMI^m	*Goals* $d_1^- = 1 - G_1 / A_1(x)$ $W_1 = 0.5$ $d_2^- = 1 - A_2(x) / G_2$ $W_2 = 0.5$
Minimize	*Minimize*
Deviation from target	$Z = W_1 d_1^- + W_2 d_2^-$

Since the beam at Level 2 does not have constant material properties, the beam performance equations presented in Equation 4.2 – Equation 4.5 cannot be used. Therefore, to enable heterogeneous material properties in the resultant beam, the beam is modeled in COMSOL, a finite element modeling software. A flow-chart of the modeling techniques used in beam design at Level 2 is shown in Figure 4.9.

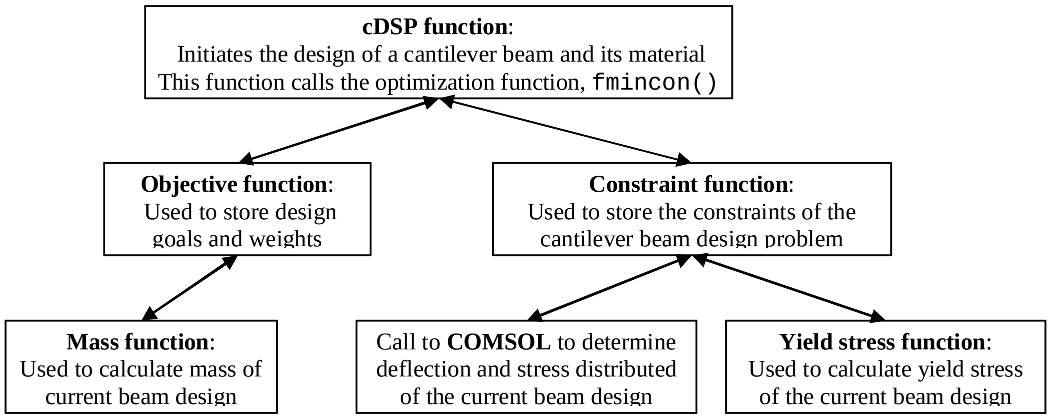


Figure 4.9 – Information flow in beam design at Level 2

The beam is modeled in COMSOL using ten discrete segments with independent material properties, as shown previously in Figure 4.4. COMSOL is chosen because of its availability and ease of use; COMSOL is compatible with MATLAB. The volume fraction (*vf*) in each of the segments are assigned independently. The inputs to the COMSOL beam performance model are the characteristic dimension of the beam cross-section (*a*) and the volume fraction in each segment (*vf1 – vf10*). The characteristic dimension (*a*) is held constant for all sections to avoid stress concentrations. The outputs of the interaction model are the maximum deflection of the beam, the maximum stress, and the location of maximum stress. A simple script to calculate the mass of the beam is also created in MATLAB by summing the mass of the individual segments based on the volume fractions in each segment and the densities of the alloy components, as shown in Equation 4.7. Equations for variation in mass and HD-EMI$_{mass}$ are also given.

$$m_{total} = 0.2a^2\left(\sum_{i=1}^{10} \rho_{steel} + vf_i\left[\rho_{al} - \rho_{steel}\right]\right) \tag{4.7}$$

$$\Delta m = \left|\frac{\partial m}{\partial \rho}\right| \Delta \rho = a^2 L \Delta \rho \tag{4.8}$$

$$\mathrm{HD-EMI}_{mass} = \min\left(\frac{\left|mean_{mass} - b_{j,mass}\right|}{\left|mean_{mass} - b_{j,mass}^{mass}\right|}\right) \tag{4.9}$$

Once the deflection and stress of the beam are calculated in COMSOL, this information is passed to a MATLAB file implementing the built-in optimization function `fmincon()` (MATLAB 2004). Similar to Level 1, beam performance information, combined with specified constraints and goals are used to determine a beam solution that is robust to variation in material properties.

Level 3

In cantilever beam Level 3 analysis, the material properties of the beam are continuously changing along the length of the beam. In order to analyze the performance of a functionally graded cantilever beam, the design optimization capabilities of MATLAB are combined with the beam response measurements obtained from FEA software COMSOL (similar to the design approach for Level 2, Figure 4.9). In COMSOL, the cantilever beam is modeled as a single part with material properties based on continuous functions. Using COMSOL, material properties can be defined based on a set of control points that are joined using a series of connected cubic splines. In the cantilever beam design process, 21 control points are defined along the length of the beam (*cp1 – cp 21*) as shown in Figure 4.10. The control points are connected with smooth, continuous functions mathematically defined by a series of 20 joined cubic splines. The control points are design variables that describe the volume fraction of the material at a particular location along the beam. Therefore, by changing the control points, the material properties and performance of the beam change.

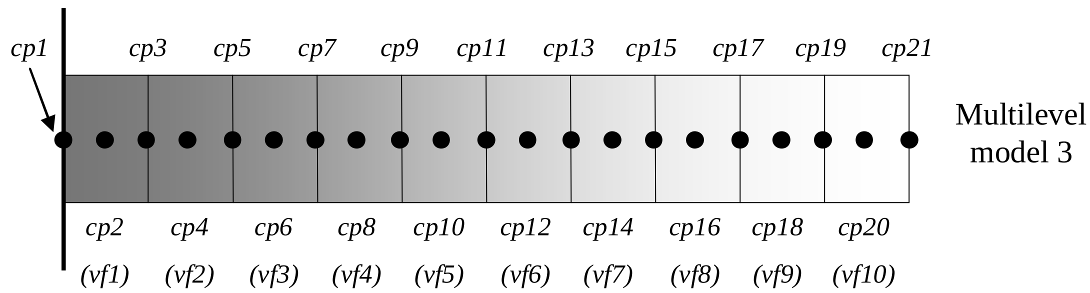


Figure 4.10 – Cantilever beam design approach for Level 3

Similar to design at Level 1 and Level 2, the MATLAB minimization function `fmincon()` is used to determine a robust design solution. However, with 22 design variables in Level 3 (cp1 – cp21, *a*) the design space cannot be fully explored due to the limitations of computing software. Therefore, the number of design variables is reduced from 21 to 12 by assuming that the values for *cp2, cp4, cp6, ... , cp20* are equal to the values of *vf1 – vf10* determined in cantilever beam design at Level 2, shown in Figure 4.11. In addition to reducing the number of design variables (without taking away from the design complexity) this method transfers design information from Level 2 to Level 3, an important step in inductive multilevel design, discussed in a later section. With a functionally graded cantilever beam, calculated beam mass is no longer trivial. The exact formula for beam mass is given in Equation 4.10.

$$m_{total} = a^2 \int_{x=0}^{x=L} \rho(x) dx \tag{4.10}$$

For this design problem, the equation for beam density consists of a series of 20 cubic equations in which each equation defines a function for beam density along 0.1 meters of the beam. Due to this complexity, an approximation is used to determine beam mass, shown in Equation 4.11. Equations for variation in mass and HD-EMI$_{mass}$ are also given.

$$m_{total} = 0.1a^2 \left(\sum_{i=1}^{20} \frac{\rho_i + \rho_{i+1}}{2} \right) \quad (4.11)$$

$$\Delta m = \left| \frac{\partial m}{\partial \rho} \right| \Delta \rho = a^2 L \Delta \rho \quad (4.12)$$

$$\text{HD} - \text{EMI}_{mass} = \min \left(\frac{\left| mean_{mass} - b_{j,mass} \right|}{\left| mean_{mass} - b_{j,mass}^{mass} \right|} \right) \quad (4.13)$$

A cDSP describing the cantilever beam design process at Level 3 is shown in Table 4.7.

Table 4.7 – Cantilever beam Level 3 cDSP

Word Formulation	**Mathematical Formulation**
Given	*Given*
Cantilever beam with a square cross-section loaded at the free end with beam weight modeled as a distributed load	Force at free end: F = 10 N Beam length: L = 2 m Beam cross-section: $A = a^2$
Material properties reflect a steel / aluminum alloy using rule of mixtures where *vf* is percent of aluminum present in alloy	$X_{alloy} = (vf)X_{Al} + (1 - vf)X_{Steel}$
Continuous material properties defined by 21 control point connected with a series of cubic splines	
Material property uncertainty model	$\Delta vf = 0.1$
Cantilever beam performance models: deflection mass safety factor	 $\delta = f(F, L, a, \rho, E)$ $m = f(a, L, \rho)$ $S.F. = f(F, L, a, \rho, \sigma_y)$
Find	*Find*
Beam design variables	Dimensions of beam cross-section: (a) Material properties of beam: (*vf1*, *vf3*, *vf5*, ... , *vf21*)
Beam deviation variables	d_i^+, d_i^-
Satisfy	*Satisfy*
Constraints Maximum allowable deflection Minimum allowable safety factor Multilevel inductive constraint	*Constraints* $\delta + \Delta\delta \leq 1$ cm s.f. ≥ 1 $cp_{2i,level3} = vf_{i,level2}$ $i = \{1,2,3,\ldots,10\}$

Table 4.7 (continued) – Cantilever beam Level 3 cDSP

Bounds volume fraction density yield strength elastic modulus cross-section dimension deviation variables	*Bounds* $0 \leq vf \pm \Delta vf \leq 1$ $2700 \text{ kg/m}^3 \leq \rho \leq 7850 \text{ kg/m}^3$ $105 \text{ MPa} \leq \sigma_y \leq 325 \text{ MPa}$ $69 \text{ GPa} \leq E \leq 205 \text{ GPa}$ $0.5 \text{ cm} \leq a \leq 15 \text{ cm}$ $d_i^+, d_i^- \geq 0$ $d_i^+ \cdot d_i^- = 0$
Goals Minimize mass (m) Maximize HD-EMI^m	*Goals* $d_1^- = 1 - G_1 / A_1(x) \quad W_1 = 0.5$ $d_2^- = 1 - A_2(x) / G_2 \quad W_2 = 0.5$
Minimize	*Minimize*
Deviation from target	$Z = W_1 d_1^- + W_2 d_2^-$

Feasible Design Space

The feasible design space at each level of design complexity is defined by bounds placed on design variables. That is, it is impossible to achieve a design solution outside the design variable bounds. In beam design, bounds on design material property variables are determined based on material properties of known materials (steel and aluminum). Design bounds for beam cross-section dimension (a) are based on estimations of beam manufacturability. Bounds placed on volume fraction / control point measurements span from 0% to 100% representing the percentage of aluminum present in the steel / aluminum alloy used for beam design. A summary of the feasible design space for multilevel cantilever beam design is given in Table 4.8.

Table 4.8 – Bounds on design variables for cantilever beam desgin

Design Variable	Lower Bound	Upper Bound
a	0.05 m	0.15 m
ρ	2700 kg/m^3	7850 kg/m^3
E	69 GPa	205 GPa
σ_y	105 MPa	325 MPa
vf / cp	0 %	100 %

Map

Mapping functions are created in order to link design information (design variable information and design uncertainty information) among design levels. In the multilevel design template developed in Chapter 3, mapping functions are used to: map discrete points (at level $i + 1$) to solution ranges (at level i), then to map solution ranges (at level i) to a robust solution (at level $i + 1$). For the cantilever beam example problem, mapping functions are created to describe the relationship of material properties and uncertainty models when traveling from one multilevel model to another. These mapping functions are mathematical descriptions of material and uncertainty information propagation in a multilevel design process. Deductive and inductive material property mappings and uncertainty mappings for cantilever beam design are presented in the following section.

Material Mapping Functions

A visual representation used to describe material property mappings among the three levels in the cantilever beam design problem is given in Figure 4.11 and Figure 4.12. The material mappings are applied to all material properties used in beam performance analysis and include density (ρ), elastic modulus (E), and yield strength (σ_y).

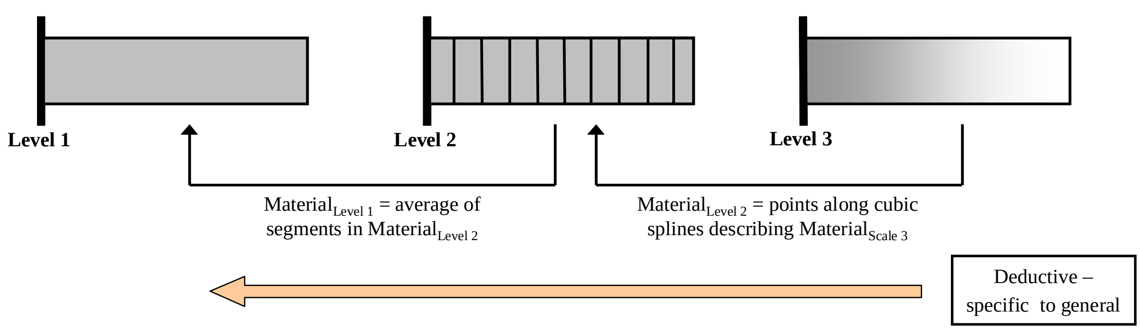


Figure 4.11 – Deductive material mapping functions for cantilever beam desgin

A description of the specific mapping functions follows:

Deductive mapping functions

- *Level 3 to Level 2* – In Level 3, a series of cubic splines are used to define material properties in the cantilever beam as a function of position along the length of a beam. In order to determine material properties in the discrete segments of a cantilever beam in Level 2, the continuous material functions are evaluated at specific locations along the length of the beam.
- *Level 2 to Level 1* – The homogeneous material properties in Level 1 are defined as the average of the heterogeneous material properties of Level 2.

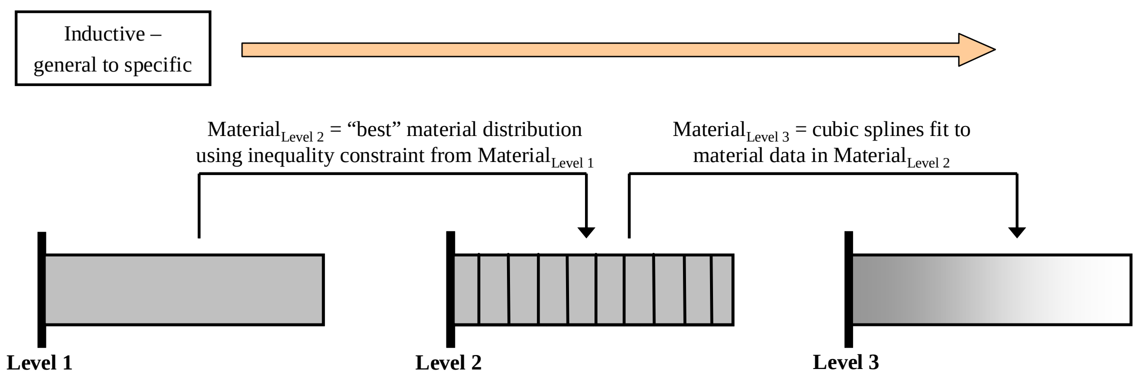


Figure 4.12 – Inductive material mapping functions for cantilever beam design

Inductive mapping functions

- *Level 1 to Level 2* – When solving for a robust beam design at Level 2, an additional constraint is added such that the average material properties at Level 2 are within 5% of the material properties solved in Level 1. This constraint preserves design information from Level 1 when making design decisions at Level 2 (Equation 4.17). A 5% error bound is allowed so that the design problem is not overly restrictive and a feasible solution can be found.
- *Level 2 to Level 3* – The 10 design variables describing volume fraction in Level 2 (*vf1* – *vf10*) are preserved in the Level 3 robust design solution.

Material property functions at Level 3 are modeled as a series of cubic splines. The volume fraction solved for in Level 2, in addition to 11 other design points, are used as control points in defining material property cubic splines at Level 3 (Equation 4.18).

Uncertainty Mapping Functions

In order to determine a robust cantilever beam design, uncertainty in the beam example problem must be identified and modeled. It is assumed that material properties of the beam are uncertain design parameters. The material properties of interest in this design problem are density (ρ), elastic modulus (E), and yield strength (σ_y). Each material property can also be described using the volume fraction (vf) calculation. For the cantilever beam example problem, it is assumed that each material property contains a 10% error bound. That is, when specifying a material property in the design process, the uncertainty in the material property is modeled as bounds surrounding the designed material property value. A representation of this concept is shown in Figure 4.13.

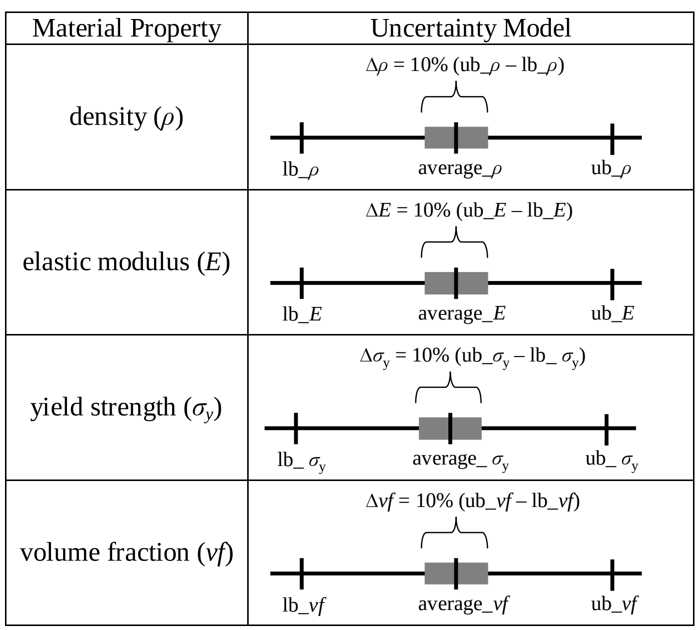


Figure 4.13 – Uncertainty mapping of cantilever beam material properties

Find

Design variables and deviation variables at each level of design are determined for the robust solution, and are listed in Table 4.9.

Table 4.9 – Design variables and deviation variables to find in beam design solution

Design Level	Design Variables	Deviation Variables
Level 1	*vf, a*	d_i^+, d_i^-
Level 2	*vf1 – vf10, a*	d_i^+, d_i^-
Level 3	*cp1 – cp21, a*	d_i^+, d_i^-

Satisfy

Constraints

For cantilever beam design, performance constraints are given in Equation 4.14 and Equation 4.15. The maximum allowable beam deflection is 1 cm, and the safety factor at any location throughout the beam must be greater than or equal to 1.

$$\delta_{\max} \leq 1\ cm \tag{4.14}$$

$$\text{safety factor} \geq 1 \tag{4.15}$$

Because of the uncertainty associated with material properties, an additional constraint must be added to the solution-finding algorithm. Since the material properties are uncertain up to 5% above and below the mean predicted value, the material properties determined for the solution must not vary outside the lower bound and upper bound set for each material property. This constraint is represented in Equation 4.16.

$$
\begin{aligned}
&(\text{lower bound}) - \left(x - \frac{1}{2}\Delta x \right) \leq 0 \\
&\left(x + \frac{1}{2}\Delta x \right) - (\text{upper bound}) \leq 0 \qquad (4.16) \\
&\forall x = \{\rho, E, \sigma_y\}
\end{aligned}
$$

In an inductive design process, an additional constraint is imposed such that the robust design solution at level i + 1 is constrained to behave similar to the robust design solution previously determined at level i. Linking design levels in inductive design through imposed design constraints is given in Equation 4.17 (Level 1 to Level 2) and Equation 4.18 (Level 2 to Level 3).

$$
\frac{\sum_{i=1}^{10} vf_{i,level2}}{10} = (\pm 5\%) vf_{level1} \qquad (4.17)
$$

$$
\begin{aligned}
&cp_{2i,level3} = vf_{i,level2} \\
&i = \{1,2,3,\ldots,10\}
\end{aligned} \qquad (4.18)
$$

<u>Goals</u>

The two goals of the cantilever beam design problem are to minimize the mass of the beam, and to maximize the robustness of beam with respect to variation in material properties. The metric used for determining design robustness is HD-EMI. In practical terms, the HD-EMI metric is a measure of the distance from design space bounds divided by system performance variation. Since there is only one performance goal in the cantilever beam design problem (minimize mass), the HD-EMI calculation only considers variation of mass in robustness calculations. In Equation 4.19 – Equation 4.21 HD-EMI calculation as it relates to design variables and performance goals at each design level is given. Notice that by increasing distance from feasible design space and decreasing variation in system performance, one is able to maximize HD-EMI.

$$\mathrm{HD-EMI}_{mass,level1} = \frac{\min\left(\left|x - x_{boundary}\right|\right)}{\max\left(\Delta mass\right)} \qquad \forall x = \{vf, a\} \tag{4.19}$$

$$\mathrm{HD-EMI}_{mass,level2} = \frac{\min\left(\left|x - x_{boundary}\right|\right)}{\max\left(\Delta mass\right)} \qquad \forall x = \{vf1 - vf10, a\} \tag{4.20}$$

$$\mathrm{HD-EMI}_{mass,level3} = \frac{\min\left(\left|x - x_{boundary}\right|\right)}{\max\left(\Delta mass\right)} \qquad \forall x = \{cp1, cp3, cp5, \ldots, cp21, a\} \tag{4.21}$$

Minimize

A deviation function in minimized in order to determine a design solution that best meets design goals. In this thesis, due to its ease of use, the Archimedian formulation of the deviation function is chosen. It consists of a weighted sum of the deviation variables, and the weights are chosen to reflect designer preferences such that they are all greater than or equal to zero and sum to unity.

Weighted Sum of Deviation Variables

The deviation function for the BRP design is shown in Equation 4.22. The goal is to minimize the value of Z in order to find the design point with the smallest deviation from design goals. In cantilever beam design, each goal is weighted equally at $W_i = 0.5$ ($i = 1, 2$).

$$Z = W_1 \cdot d_1^- + W_2 \cdot d_2^- \quad (4.22)$$

4.2.3 Cantilever Beam Inductive Design Solution

The inductive multilevel robust cantilever beam design solution is presented in Section 4.3.3. Following a general-to-specific (inductive) approach, the design solutions are presented in a top-down manner for Level 1, Level 2, and Level 3.

Level 1

When obtaining an inductive multilevel design solution, the designer begins at Level 1, the least amount of design complexity. The cantilever beam design solution and performance data at design Level 1 are presented in Table 4.10 – Table 4.12.

Table 4.10 – Cantilever beam design variable data: Level 1

Design Variable	Value	Units
a	11.3	cm
vf	0.5	none

Table 4.11 – Cantilever beam performance data: Level 1

Performance	Value	Units
max deflection	0.1	cm
mass	134.8	kg
variation of mass	13.1	kg
safety factor	1	none

Table 4.12 – Cantilever beam material property data: Level 1

Material Property	Value	Units
density	5275	kg/m^3
elastic modulus	137	GPa
yield strength	215	MPa

Level 2

Once the robust design solution is obtained for Level 1, an additional design constraint characterizing cantilever beam design at Level 1 is imposed for cantilever beam design at Level 2. The cantilever beam design solution and performance data at design Level 2 are presented in Table 4.13 – Table 4.15.

Table 4.13 – Cantilever beam design variable data: Level 2

Design Variable	**Value**	**Units**
a	2.9	cm
vf1	0.12	none
vf2	0.36	none
vf3	0.48	none
vf4	0.48	none
vf5	0.54	none
vf6	0.54	none
vf7	0.54	none
vf8	0.66	none
vf9	0.66	none
vf10	0.94	none

Table 4.14 – Cantilever beam performance data: Level 2

Performance	**Value**	**Units**
max deflection	1.0	cm
mass	8.3	kg
variation of mass	0.8	kg
min safety factor	11.86	none

Table 4.15 – Cantilever beam material property data: Level 2

Material Property				
Segment	**Location (meters)**	**Density (kg/m3)**	**Elastic Modulus (GPa)**	**Yield Strength (MPa)**
1	0 to 0.2	7225	188	298
2	0.2 to 0.4	5977	156	245
3	0.4 to 0.6	5381	140	220
4	0.6 to 0.8	5361	139	219
5	0.8 to 1.0	5086	132	207
6	1.0 to 1.2	5086	132	207
7	1.2 to 1.4	5086	132	207
8	1.4 to 1.6	4440	115	179

Table 4.15 (continued) – Cantilever beam material property data: Level 2

9	1.6 to 1.8	4439	115	179
10	1.8 to 2.0	2989	77	117

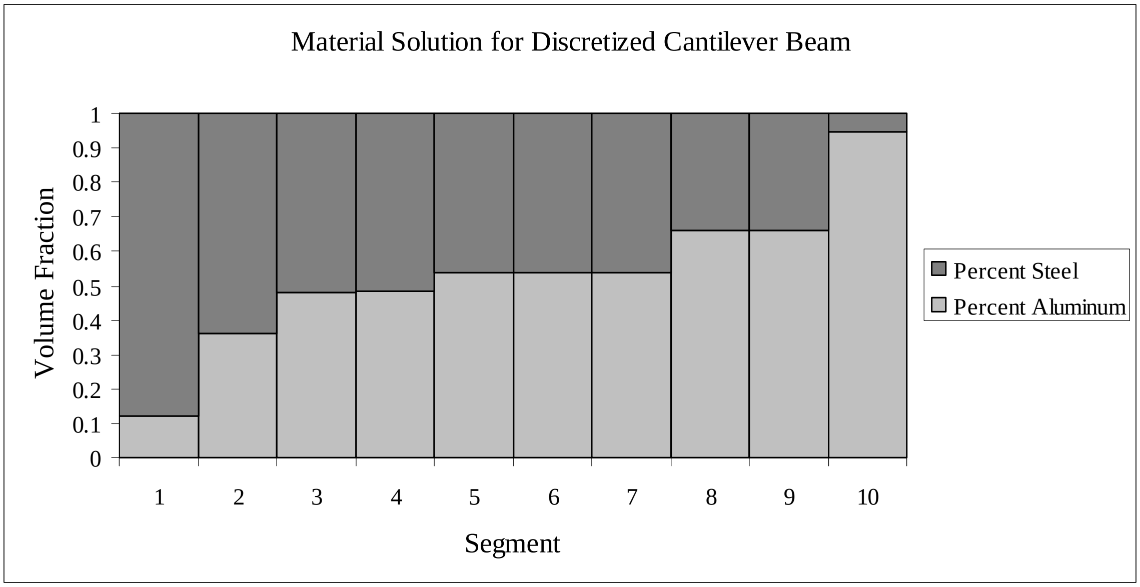


Figure 4.14 – Volume fraction graph for Level 2 beam design

Level 3

Once the robust design solution is obtained for Level 2, an additional design constraint characterizing cantilever beam design at Level 2 is imposed for cantilever beam design at Level 3. The cantilever beam design solution and performance data at design Level 3 are presented in Table 4.16 – Table 4.18. Based on the complexity requirements of the cantilever beam design problem, design solutions at Level 3 are considered the final multilevel design solution.

Table 4.16 – Cantilever beam design variable data: Level 3

Design Variable	**Value**	**Units**
a	2.6	cm
cp1	0.07	none
cp3	0.24	none

Table 4.16 (continued) – Cantilever beam design variable data: Level 3

cp5	0.36	none
cp7	0.49	none
cp9	0.50	none
cp11	0.55	none
cp13	0.57	none
cp15	0.60	none
cp17	0.60	none
cp19	0.94	none
cp21	0.94	none

Table 4.17 – Cantilever beam performance data: Level 3

Performance	**Value**	**Units**
max deflection	1.0	cm
mass	6.9	kg
variation of mass	0.7	kg
min safety factor	9.50	none

Table 4.18 – Cantilever beam material property data: Level 3

Material Property				
Segment	**Location (meters)**	**Density (kg/m^3)**	**Elastic Modulus (GPa)**	**Yield Strength (MPa)**
1	0 to 0.1	7473	195	309
2	0.1 to 0.2	7225	188	298
3	0.2 to 0.3	6638	173	273
4	0.3 to 0.4	5977	156	245
5	0.4 to 0.5	6005	156	246
6	0.5 to 0.6	5381	140	220
7	0.6 to 0.7	5303	138	216
8	0.7 to 0.8	5361	139	219
9	0.8 to 0.9	5295	138	216
10	0.9 to 1.0	5086	132	207
11	1.0 to 1.1	5014	130	204
12	1.1 to 1.2	5086	132	207
13	1.2 to 1.3	4936	128	201
14	1.3 to 1.4	5086	132	207
15	1.4 to 1.5	4779	124	194
16	1.5 to 1.6	4440	115	179
17	1.6 to 1.7	4779	124	194
18	1.7 to 1.8	4439	115	179
19	1.8 to 1.9	3029	78	119
20	1.9 to 2.0	2989	77	117

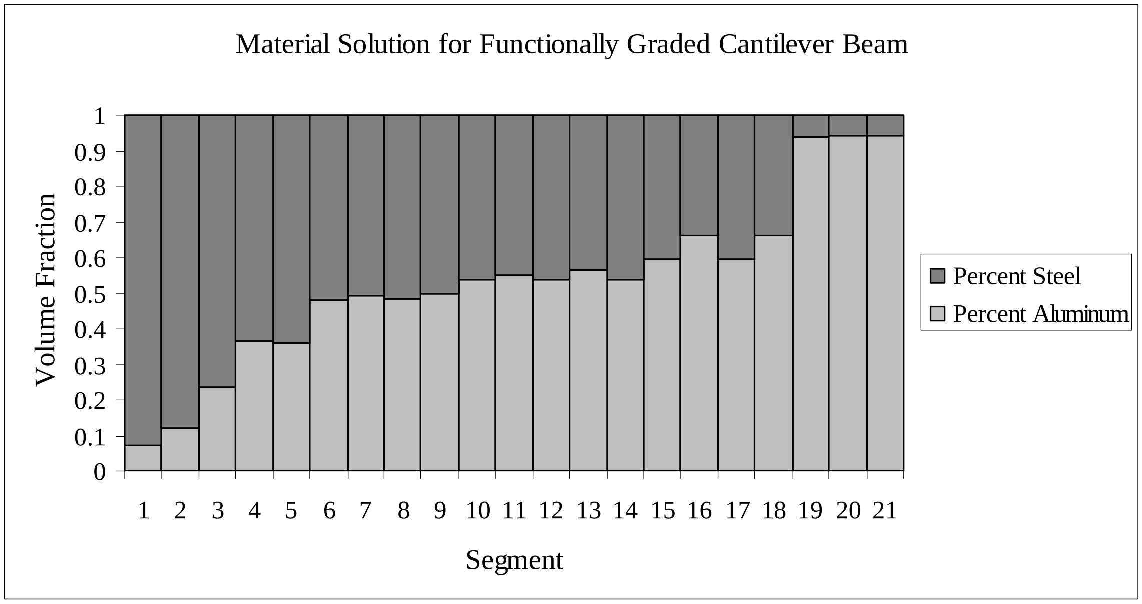
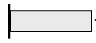
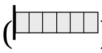


Figure 4.15 – Volume fraction graph for Level 3 beam design

4.3 VERIFICATION AND VALIDATION BASED ON CANTILEVER BEAM DESIGN

The following section contains evidence for the verification and validation of the template-based approach to multilevel robust design presented in Chapter 3 by considering the design of a cantilever beam and its associated material. First, the validity of the design solution is examined. Then, the results obtained from completing the BRP example problem are discussed in terms of validating the proposed template-based approach to multilevel systems design.

4.3.1 Verification and Validation of Computational Design Tools in Cantilever Beam Design

In the following section, the validity of the computational design tools used in the cantilever beam example problem is examined. In order to reach design solutions in the cantilever beam example problem, performance models and finite element analysis are employed. Fundamental beam deflection equations (Euler's beam equations) are used to determine the deflection of a beam with homogeneous material properties under constant loading (Level 1, ). When the material properties of the beam are not constant, Euler's beam equations can no longer be used to predict beam deflection. Therefore, for cantilever beam design at Level 2 () the discretized beam is modeled in the FEA software COMSOL where beam deflection and stress are measured. The functionally graded cantilever beam (Level 3, ) is also modeled in COMSOL. Continuous functions describing the material properties of the beam are defined by a set of control points connected by cubic splines. Similar to analysis at Level 2, beam deflection and stress are determined using COMSOL for the functionally graded beam at Level 3. For cantilever beam design at all levels, the design problem is formulated as a compromise Decision Support Problem with design variables, constraints, goals, and preferences. The cDSP at each level is solved in MATLAB using the solution-finding algorithm, `fmincon()` (MATLAB 2004).

4.3.2 Verification and Validation of Multilevel Design Template Based on Cantilever Beam Design

One of the main goals in completing the cantilever beam example problem is to provide evidence in the verification and validation of the developed template for multilevel robust design. Recall the Validation Square discussed in detail in Chapter 2. To summarize, the Validation Square, composed of four sections, is a construct used in the systematic verification and validation of design methods. Recall that the sections of the Validation Square dealing with the application of the proposed design method to example problems include *domain-specific structural validity* and *domain-specific performance validity*. In the following sections, ways in which completing the cantilever beam example problem adds value to the domain-specific structural validity and domain-specific performance validity of the developed multilevel design template are presented.

Domain-Specific Structural Validity

Domain-specific structural validity relates to the appropriateness of the selected example problem and the designer is prompted to ask the question, "Is the example problem used in demonstrating the method an appropriate choice?" It is asserted that the cantilever beam example problem is an appropriate choice for testing the effectiveness of the developed multilevel design template for because the design problem possesses the following characteristics:

- *Clearly defined design problem* – The cantilever beam design problem contains a clearly defined problem statement with specified design variables, bounds, constraints, goals, and preferences (formulated in a cDSP). Each of these descriptions is needed for the successful implementation of the multilevel design template. Additionally, material property mappings and uncertainty mappings are known or easily determined for the cantilever beam design problem. The multilevel robust design temple is developed for a design environment in which design requirements, bounds, constraints, goals, and preferences are clearly known.
- *Multilevel design problem* – The cantilever beam problem is inherently multilevel in nature because it is used to examine the design of a product and material. The various levels of model complexity are used to describe the design of the material in more specific detail. The various levels in the cantilever beam design problem are used to capture more complex details of the performance of the product based on material properties. Only three levels are considered in this design problem due to computational and time constraints. However, to increase the value in the materials design portion of this example problem, model of increased complexity (such as models that capture the micro- and nano- nature of the material) should be considered. Due to the multilevel nature of the cantilever beam example

problem it is an appropriate choice for applying the developed multilevel design template presented in Chapter 3.

Domain-Specific Performance Validity

Domain-specific structural validity relates to the outcome of applying the method to an example problem and is used to ask the question, "Does the application of the method to the example problem produce useful results?" To adequately address this question two topics are considered: the usefulness of the numerical design solution and the overall usefulness of the multilevel design template.

One way to analyze the effectiveness of the numerical results obtained from the cantilever beam design problem is to compare numerical solutions with expected solution trends. It is expected that the strongest (and heaviest) material would appear at the base of the beam where stress is the greatest. It is also assumed that moving from the base to the free end of the beam the strength (and density) of beam material decreases. This trend is observed in the cantilever beam design solutions at all levels. The agreement with numerical results and expected trends builds confidence in the validity of the cantilever beam design solution.

The internal consistency of the numerical design solution is also tested with a starting point analysis. The cantilever beam example problem is solved using an optimization routine at each design level. Therefore, it is important to determine if the selected starting point at each level results in a robust, stable solution that most closely meets design goals. For cantilever beam design at Level 1, ten different starting points spanning the range of all possible design variable values result in identical design solutions and beam performance. Therefore, it is concluded that the cantilever beam design solution at Level 1 is robust to changes in starting point, and the best solution at

Level 1 is identified. Cantilever beam design at Level 2 and Level 3 requires the use of computationally expensive finite element software. Also, the number of design variables at Level 2 and Level 3 increase the probability that implementing the optimization routine will result in a local (and not global) minimum. Due to high computation cost for each design run at Level 2 and Level 3, a slightly different approach is taken in order to ensure a reasonable solution is reached. The starting point at Level 2 and Level 3 is set to reflect the expected solution trend with the strongest (and heaviest) material at the base of the beam, and material strength (and density) decreasing along the length of the beam. By beginning the optimization routine in this way, it is found that more reasonable, logical solutions are obtained that reflect the expected trends of cantilever beam design.

Additionally, confidence is built in the domain-specific performance validity of the multilevel design template based on its overall usefulness in the cantilever beam design problem. First, by applying the multilevel design template, the cantilever beam design problem is divided into levels of varying model complexity. That is, by using the multilevel design template, the computationally rigorous task of designing a product and material is partitioned into more manageable design problems at increasing levels of model complexity. Applying the design template also provides a guided direction for the designer to proceed through the design process. Additionally, information collected during the design process is stored in the design template and organized for future design space exploration in cantilever beam design. The modular nature of the design template allows for expanding aspects within the cantilever beam design problem, and / or combining the cantilever beam design problem with other design processes. In addition to providing the overall construct to facilitate multilevel robust design, by implementing the design template, a designer is required to develop and utilize mapping functions used to travel (deductively and inductively) through levels of design complexity. The developed mapping functions are useful tools facilitating low-cost design space exploration at various levels of model complexity. Finally, applying the multilevel design template guides the designer to the determination of a multilevel robust design solution. The achieved solution meets design constraints, and as design complexity increases, design goals are more closely met. The inductive design solution also agrees with what is intuitively expected—the beam material is strongest at the base and decreases in strength (and density) as one moves from the base to the free end.

To summarize, the template-based approach to multilevel robust cantilever beam design is a valuable design strategy because the complexity of the design problem is decreased leading to agile design space exploration, design information is store in a organized and modular fashion for future design exploration, and at low computation cost, mapping functions are used to travel throughout the multilevel design problem in both a top-down and bottom-up path. A visual representation of the value added to the verification and validation of the developed design template provided in Chapter 4 is shown in Figure 4.16.

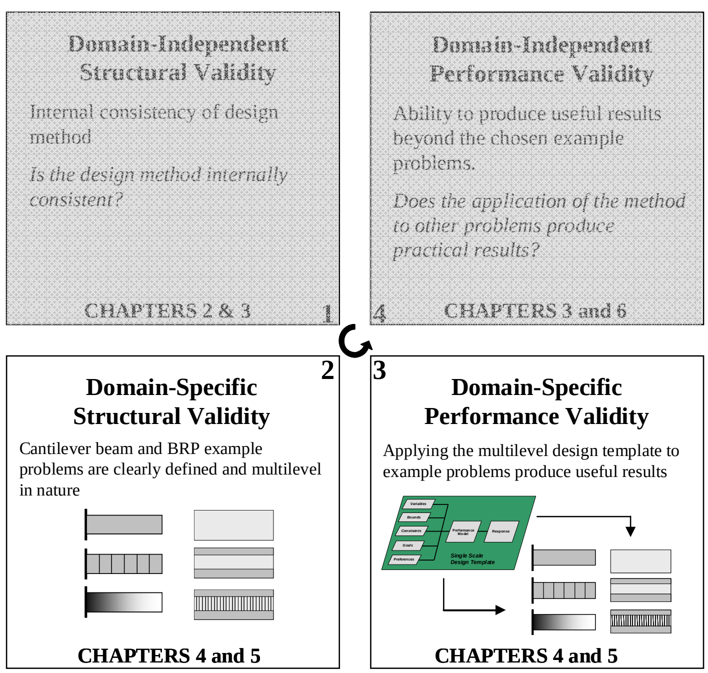


Figure 4.16 – Value added to verification and validation of design template – Chapter 4

4.4 SYNOPSIS OF CHAPTER 4

The completion of the cantilever beam design problem adds value to the validation of the multilevel design template. The multilevel design template is used to guide the designer through a multilevel inductive design process, and useful design solutions are obtained. Recall from Chapter 1 and Chapter 2 that the context for this thesis is set with a discussion of multilevel design, design uncertainty, robust design, and template-based design. Then, in Chapter 3, the multilevel design template is presented. In Chapter 4 and Chapter 5, the template is applied to example problems in order to determine its domain-specific structural and performance validity. Chapter 6, concludes this thesis with a summary of the validation of the multilevel design template based on the Validation Square. The overall value of a template-based approach to multilevel design as well as the intellectual contributions of this thesis is presented in Chapter 6.

CHAPTER 5

COMPREHENSIVE EXAMPLE – DESIGN OF A BLAST RESISTANT PANEL

In order to demonstrate the use of the multilevel design template presented in Chapter 3, a comprehensive example problem is completed. In Chapter 5, the design of a blast resistant panel is investigated. Blast resistant panels (BRPs) are sandwich structures designed to experience less deflection under blast loading compared to solid panels of equal mass. The blast resistant panels designed in this thesis are layered panels consisting of a front face sheet, core, and back face sheet. The front and back face sheets are solid panels, whereas the core is a honeycomb cellular structure, which dissipates large amounts of impulse energy due to core crushing. Based on proposed BRP military armor applications, the design goals are to minimize BRP deflection, minimize BRP mass / area, and maximize BRP robustness with respect to uncertainty in material properties and loading conditions.

In this thesis, BRP design is divided into levels according to model complexity. Specifically, the BRP example problem is divided into 3 levels of model complexity: BRP modeled as two solid panels surrounding a honeycomb core [Level 3, greatest complexity], BRP modeled as three solid panels [Level 2, moderate complexity], and BRP modeled as a single solid panel [Level 1, least complexity]. Similarly to Chapter 4, BRP multilevel design is explored using models of increasing complexity (Level 1 to Level 3). After multilevel BRP performance models are developed, mapping functions describing material property and uncertainty relationships among the various levels are discussed. Then, an inductive BRP design solution is presented and discussed. Chapter 5 concludes with a discussion of the advantages of a template-based approach to multilevel design, and how completing the multilevel robust design of BRPs adds value to the verification and validation of the template-based approach to the robust design of multilevel systems. The complex nature of the BRP example problem builds confidence in the likelihood that the multilevel design template can be successfully applied to a variety of complex engineering design problems. A summary of the information in Chapter 5 is given in Table 5.1 and Figure 5.1 illustrates how Chapter 5 is connected to other ideas in this thesis.

Table 5.1 – Summary of Chapter 5

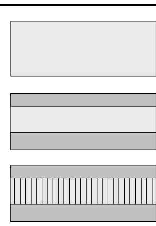
Heading / Sub-Heading	**Information**
Problem Overview	
Introduction to Example Problem	BRP example problem is introduced including: - Design requirements (included in Appendix B) - Design goals
Design Approach	Nature of BRP problem is examined: - Multilevel nature of design problem - Use of design templates for achieving design solution
Value in Completing Example Problem	Address value based on: - Research questions presented in Chapter 1 - Verification and validation of a template-based approach to multilevel robust design
Design Process and Solution	
Particularization of Multilevel Robust Design Template	Multilevel design template is organized under the headings: - Given – feasible alternative, assumptions, parameters, goals - Define – design levels, feasible design space, performance metrics (additional information in Appendix C) - Map – discrete points to solution ranges (deductive), solution ranges to robust solution (inductive) - Find – design variables, deviation variables - Satisfy – constraints, bounds, goals - Minimize – weighted sum of deviation variables Multilevel template is particularized for BRP design problem
Design Process 	Multilevel nature of BRP design problem: - Level 1 – single solid panel with uniform material properties - Level 2 – three solid panels with independent material properties and layer heights - Level 3 – two solid layers surrounding honeycomb core each with independent material properties and layer thicknesses
BRP Multilevel Inductive Design Solution	Method for inductive multilevel design is presented and design solutions are obtained and discussed

Table 5.1 (continued) – Summary of Chapter 5

Verification and Validation	
Verification and Validation of Computational Design Tools	Verification and validation of the computational tools used in obtaining BRP multilevel robust design solution including: - BRP computational models (MATLAB) - BRP FEA analysis (ABAQUS) - BRP impulse loading analysis
Verification and Validation of Multilevel Robust Design Template	Value added to the verification and validation of a template-based approach to multilevel robust design based on BRP design

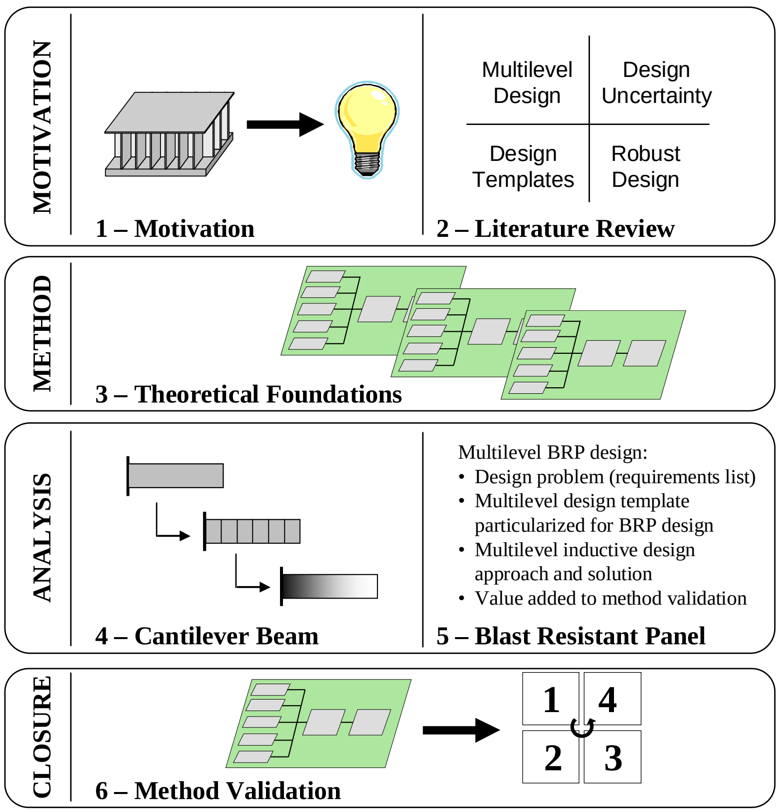


Figure 5.1 – Setting the context for Chapter 5

5.1 OVERVIEW OF COMPREHENSIVE EXAMPLE – DESIGN OF A BLAST RESISTANT PANEL

In Section 5.1 an overview of the BRP design problem is presented. Completing the BRP design problem is a collaborative effort among students in the Systems Realization Lab at Georgia Tech. The intellectual contributions from Matthias Messer, Jin Song, and

Stephanie Thompson in completing this design problem are gratefully acknowledged. A conference paper presented by Thompson and coauthors at the *AIAA/ISSMO Multidisciplinary Analysis and Optimization Conference* in 2006 details a preliminary BRP design approach and solution (Thompson, et al. 2006).

5.1.1 Introduction to Blast Resistant Panel Example Problem

The BRP example problem examined in Chapter 5 is used to demonstrate the robust design of multilevel systems using a template-based approach. BRPs are sandwich structures consisting of a front face sheet, core, and back face sheet. Under impulse loading, a BRP experiences less deflection than a similarly loaded solid plate of equal mass. The front face sheet and back face sheet of a BRP are solid plates, whereas, the core contains honeycomb cellular structures arranged perpendicular to the front and back face sheets. As a BRP is loaded perpendicular to the front face sheet, the core of the BRP collapses, absorbing large amounts of energy. Examples of BRPs with varying core topologies are shown in Figure 5.2 (Fleck and Deshpande 2004). In this thesis, BRP core topology analysis is limited to square honeycomb cores.

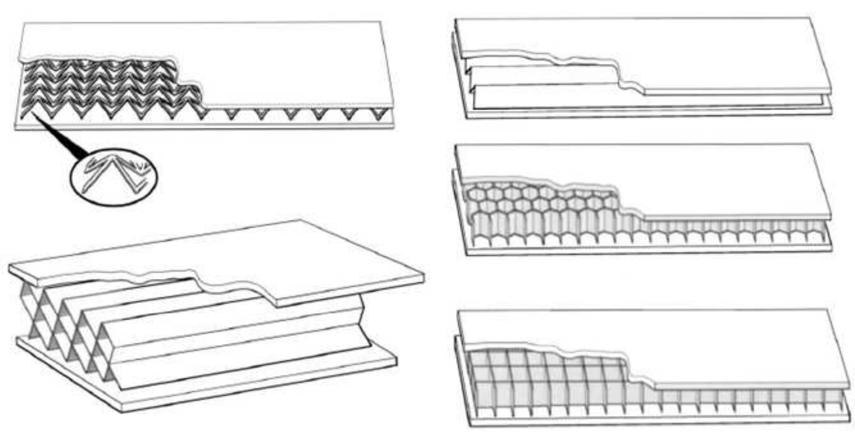


Figure 5.2 – Sample blast resistant panels (Fleck and Deshpande 2004)

For BRP design analysis in this thesis, consider a BRP consisting of two solid panels surrounding a square honeycomb core. The BRP is loaded with a spatially-uniform, time-dependent pressure [$p(t)$] along the front face sheet as seen in Figure 5.3.

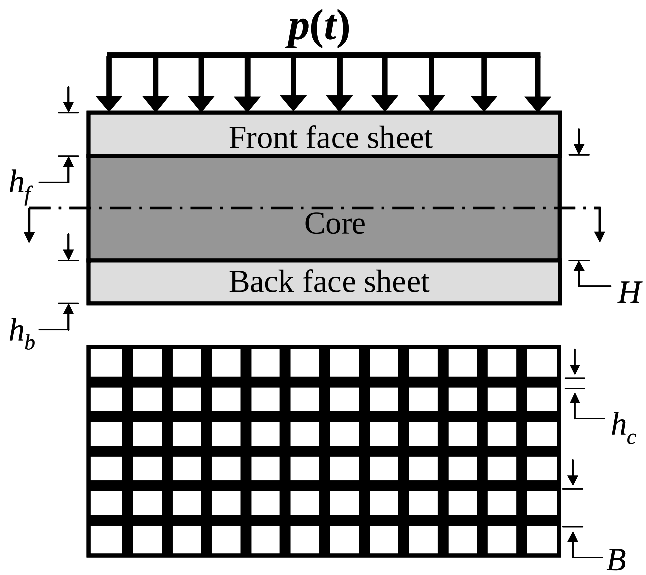


Figure 5.3 – BRP under uniform pressure loading

Noise factors and uncertain design parameters include uncertainty in loading conditions and BRP material properties. System design goals include minimizing back face sheet deflection, minimizing BRP mass / area, and maximizing BRP robustness with respect to uncertainty in material properties and loading conditions. The maximum allowable back face sheet deflection (δ_{max}) is 10% of panel length (panel length, L = 1 m), and maximum allowable mass / area (M) is 150 kg/m^2. Key BRP dimensions are given in Figure 5.3 (h_f [height of front face sheet], h_b [height of back face sheet], H [height of core], h_c [core cell wall thickness], B [core cell spacing]).

BRP performance analysis models are developed based on the work of Fleck and Deshpande, 2004 and Hutchinson and Xue, 2005. The deformation of sandwich plates under impulse loading may be divided into three time periods (Fleck and Deshpande, 2004). The three phases of panel deformation include:

- Phase 1 – blast wave encounters front face sheet
- Phase 2 – core crushing
- Phase 3 – bending and stretching of the back face sheet.

Hutchinson and Xue, 2005 adapted the three period deformation theory and applied it to the optimization of blast resistant panels. The deflection of BRPs is predicted based on calculations developed by Hutchinson and Xue. In this thesis, BRP performance models developed by Hutchinson and Xue are implemented in a template-based multilevel robust design approach.

Before a multilevel BRP design solution is obtained, a method for analyzing BRP performance at a single level is selected. A compromise Decision Support Problem (cDSP) is formulated at each design level in order to determine a robust design solution at a single level of model complexity. The cDSP is discussed in detail in Section 2.4.3. The cDSP developed at each level of design complexity is analogous to the single level design template show in Figure 3.8. A more mathematically rigorous cDSP is presented for each level of model complexity in Section 5.3.2. A cDSP for the overall BRP design problem, intended to illustrate overall design requirements and goals, is given in Table 5.2 and discussed in more detail in the following paragraphs.

Table 5.2 – cDSP for multilevel BRP design

Word Formulation	**Mathematical Formulation**
Given	*Given*
BRP with two solid layers surrounding a honeycomb core with a square topology	BRP length: $L = 1$ m
Uniform pressure impulse	Impulse load model: $p = p_0 e^{-t/t_0}$ $I_0 = \int p\mathrm{d}t = p_0 t_0$ $p_0 = 25\,\mathrm{MPa}$, $t_0 = 10^{-4}$ sec

Table 5.2 (continued) – cDSP for multilevel BRP design

Loading uncertainty models	$\Delta p_0 = 0.15\mu_{p_0}$, $\Delta t_0 = 0.15\mu_{t_0}$
Material property uncertainty models	$\Delta\sigma_y = 0.1(\sigma_{y,ub} - \sigma_{y,lb})$, $\Delta\rho = 0.1(\rho_{ub} - \rho_{lb})$
BRP performance model	$\delta = f(h, h_f, h_b, H, h_c, B, \sigma_y, \rho)$ $M = f(h, h_f, h_b, H, h_c, B, \rho)$
Find	*Find*
BRP design variables BRP deviation variables	Dimensions of BRP: (h, h_f, h_b, H, h_c, B) Material properties of each layer of BRP: (ρ_i , σ_{yi}) d_i^+, d_i^-
Satisfy	*Satisfy*
Constraints Maximum allowable deflection Maximum allowable mass / area	*Constraints* $\delta + \Delta\delta \leq 10$ cm $M + \Delta M \leq 150$ kg/m^2
Bounds density yield strength panel thickness front face sheet thickness back face sheet thickness core height cell wall thickness cell spacing deviation variables	*Bounds* 2000 kg/m^3 $\leq \rho \pm \Delta\rho \leq$ 10000 kg/m^3 100 MPa $\leq \sigma_y \pm \Delta\sigma_y \leq$ 1100 MPa 7 mm $\leq h \leq$ 100 mm 1 mm $\leq h_f \leq$ 25 mm 1 mm $\leq h_b \leq$ 25 mm 5mm $\leq H \leq$ 50 mm 0.1 mm $\leq h_c \leq$ 10 mm 1 mm $\leq B \leq$ 20 mm $d_i^+, d_i^- \geq 0$ $d_i^+ \cdot d_i^- = 0$
Goals Minimize deflection (δ) Minimize mass/area (M) Maximize HD-EMI$^\delta$ Maximize HD-EMI^M	*Goals* $d_1^- = 1 - G_1 / A_1(x)$ $W_1 = 0.25$ $d_2^- = 1 - G_2 / A_2(x)$ $W_2 = 0.25$ $d_3^- = 1 - A_3(x) / G_3$ $W_3 = 0.25$ $d_4^- = 1 - A_4(x) / G_4$ $W_4 = 0.25$
Minimize	*Minimize*
Deviation from target	$Z = W_1d_1^- + W_2d_2^- + W_3d_3^- + W_4d_4^-$

BRP Design Requirements

The BRP design requirements are inspired by a design problem currently under investigation as a joint effort between the US Army Research Lab, US Air Force, and research groups at Georgia Institute of Technology and Pennsylvania State University as part of the I/UCRC on computational materials design.

Requirements lists for completing the BRP design problem are created in order to document the functions that the designed BRP must achieve. For the BRP design problem, three requirements lists are created which reflect the areas in which the completed BRP design problem will add value. The requirements lists for the BRP design problem are divided into the following categories: advancement of multilevel robust design methodology, verification and validation of robust multilevel design template, and satisfying customer requirements.

- *Advancement of multilevel robust design methodology* – One of the goals in studying multilevel BRP design is to advance multilevel robust design methodology. By completing various example problems, researchers expect to extend current understanding of the special requirements involved in multilevel design in order to develop a detailed method for the design of multilevel systems.
- *Verification and validation of a design strategy* – In Chapter 3 theoretical foundations for a template-based approach to robust multilevel design are presented. One step in verifying the multilevel template-based design approach is to complete example problems using the developed design template. Completing example problems adds value to the domain-specific structural validity and domain-specific performance validity of method validation.
- *Satisfy customer requirements* – The BRP problem is a collaborative design project between academic, industrial, and government research organizations. Acting as the "customer", government and industrial research labs are responsible for supplying BRP performance requirements as part of a I/UCRC on computational materials design.

Requirements lists for BRP design are listed in Appendix B, Table B.1 – Table B.3. The three requirements lists correspond to the three areas of motivation for completing the BRP example problem.

BRP Design Goals

For the BRP design problem, the design goals include minimizing deflection of the back face sheet, minimizing mass / area of panel, and maximizing the robustness of the system with respect to uncertainty in loading conditions (noise factors) and material properties (uncertain control factors). Recall from Chapters 2 and 3 that HD-EMI is a metric for measuring the robustness of a system that contains uncertainty in noise factors, control factors, and propagated process chain uncertainty. By maximizing HD-EMI, a BRP design that is robust to variations in loading conditions and material composition and robust to uncertainty in the multilevel design process chain is achieved. Since there are two BRP performance goals (minimize deflection, minimize mass/area) the HD-EMI metric is maximized with respect to each design variable for each performance design goal. More details in how the HD-EMI metric is applied to the design problem at each level is given in Section 5.3.

5.1.2 Multilevel Design Approach for Blast Resistant Panel Design

In the following section, an overview of the multilevel approach used in solving the BRP design problem is presented. First, the multilevel nature of the design problem is presented. Then, a summary of the application of the multilevel design template (developed in Chapter 3) in the BRP problem is given.

BRP Design as a Multilevel Design Problem

In order to design a BRP with independent dimensions and material properties among the three layers, 11 design variables (ρ_f, ρ_b, ρ_c, $\sigma_{y,f}$, $\sigma_{y,b}$, $\sigma_{y,c}$, h_f, h_b, H, h_c, B) must be considered. Design space exploration with such a large number of design variables is difficult and costly due to the large number of design variable combinations. Therefore, it is determined that the BRP design problem can be divided into levels of models of decreased complexity, thereby reducing the number of design variables and facilitating agile design space exploration. To achieve this goal, the BRP design problem is divided into three levels of model complexity. The three levels, shown in Figure 5.4, are selected based on value added to the BRP design process and natural divisions in the problem statement.

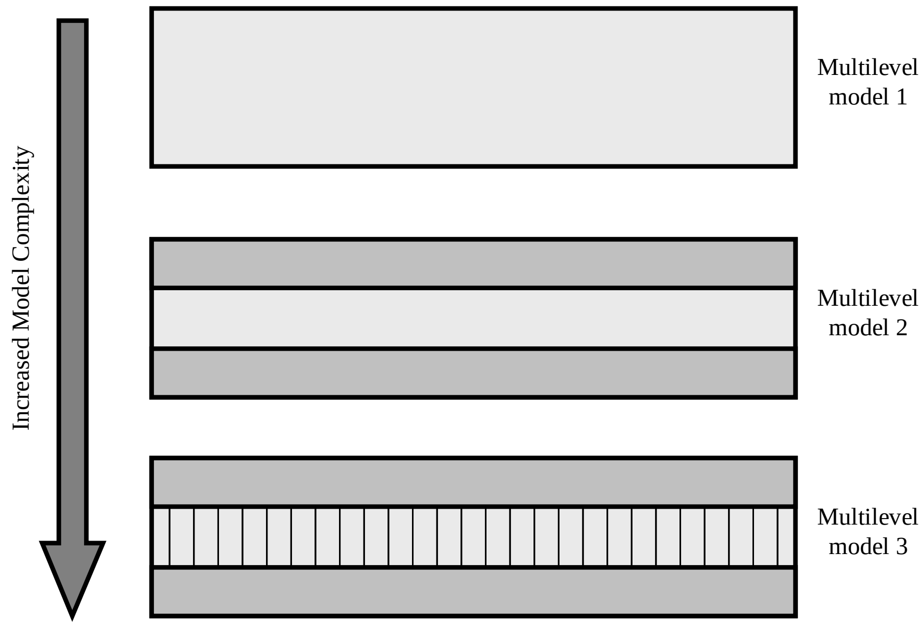


Figure 5.4 – BRP design problem divided according to levels of model complexity

First, a BRP modeled as a single solid panel is analyzed. At BRP Level 1, 3 design variables are considered (ρ [density], σ_y [yield strength], h [panel height]). Increasing in model complexity, the next level analyzed is a BRP with three solid layers (front face sheet [solid], core [solid], back face sheet [solid]). Each layer has independent material properties and thicknesses. Level 2 design analysis consists of 9 design variables (ρ_f, ρ_b, ρ_c, $\sigma_{y,f}$, $\sigma_{y,b}$, $\sigma_{y,c}$, h_f [front face sheet thickness], h_b [back face sheet thickness], H [core thickness]). Last, a BRP consisting of three layers with a honeycomb core is considered (front face sheet [solid], core [honeycomb], back face sheet [solid]). With 11 design variables, this is the most complex level considered in the BRP design problem (ρ_f, ρ_b, ρ_c,

$\sigma_{y,f}$, $\sigma_{y,b}$, $\sigma_{y,c}$, h_f, h_b, H, h_c [cell wall thickness in core], B [cell spacing in core]). In Section 5.3, the performance of a BRP is analyzed using models of varying complexity.

Application of Multilevel Robust Design Template to BRP Design

In Chapter 3 a multilevel design template is presented. This design template is applied to the cantilever beam design problem in Chapter 4 and the BRP design problem in Chapter 5. The multilevel design template presented in Chapter 3 is particularized and applied to the BRP example problem in order to facilitate the systematic design of a BRP (see Figure 5.7). In Figure 5.5, the potential for augmenting the generic multilevel design template BRP design is shown.

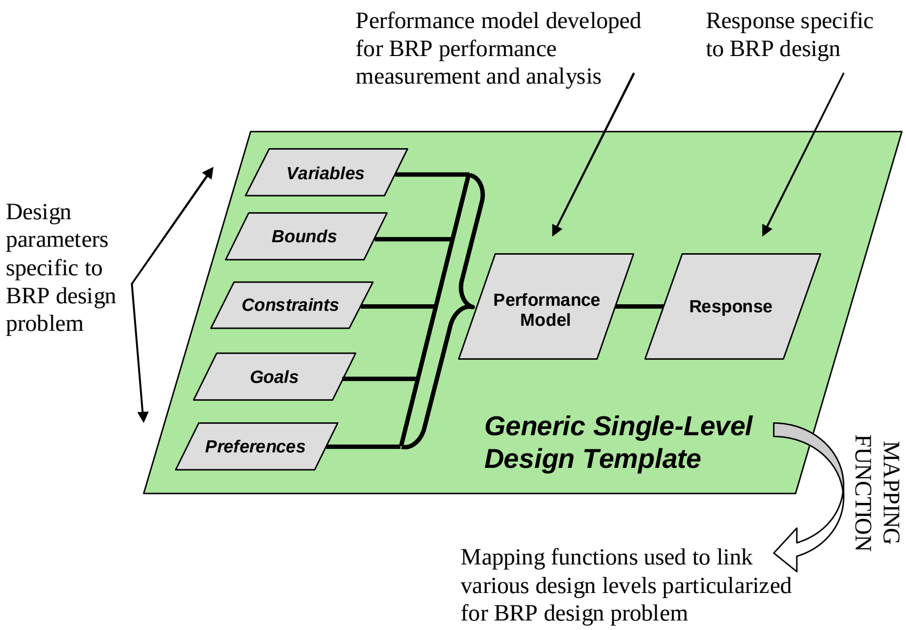


Figure 5.5 – Multilevel design approach for cantilever BRP example

For the three levels of model complexity considered in the BRP example, the generic single-level design template is particularized for the design parameters, performance model, and response at each level. Then, the single-level design templates are joined using material and uncertainty mapping functions to create the framework for inductive multilevel BRP design.

5.1.3 Value in Completing Blast Resistant Panel Example Problem

Addressing Research Questions

In the following section, the motivation for completing the BRP example problem is discussed. Motivation for completing the example problem is divided into three basic themes: to demonstrate the multilevel robust design of multilevel systems using a design template approach (in response to research questions), to provide validation of the generic multilevel design template presented in Chapter 3, and to meet customer requirements. An overview of the BRP example problem and motivation is displayed in Figure 5.6.

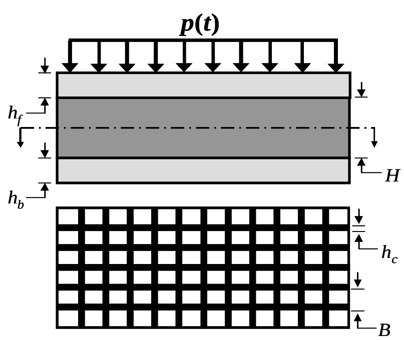
Example Problem	Motivation	
Blast Resistant Panel Design Design of a blast resistant panel 	• Design goals	minimize BRP deflection & mass maximize BRP robustness to uncertainty in material properties and loading conditions
	• Design constraints	maximum deflection $\delta_{max} \leq 0.1L$ mass / area $\leq$ 100 kg/m^2 additional constraints
	• Design variables	material properties of BRP dimensions of BRP
	• Example problem intended as a comprehensive illustration of template-based multilevel robust design and to validate developed multilevel design template	

Figure 5.6 – Overview and motivation for BRP example problem

The BRP example problem was selected because of its similarity to the topics addressed in the research questions. The primary motivation in completing the BRP example problem is to demonstrate the usefulness of a template-based approach in the robust design of multilevel systems (Research Question #1, Section 1.3). In addition, successful completion of a template-based approach to multilevel BRP design addresses themes in Research Question #2, Section 1.3.

Verification and Validation of Multilevel Robust Design Template

Additionally, the BRP design problem is chosen for investigation in this thesis because it can be used to demonstrate a template-based approach to multilevel robust design of systems, and it contains design complexities found in many design problems in academia and industry. In terms of validating the multilevel design template, the BRP example problem is useful in demonstrating the domain-specific structural validity (appropriateness of example problems) and the domain-specific performance validity (ability to produce useful results for the chosen example problems) of the developed template. The BRP example problem is also intended to illustrate the key advantages of the multilevel design template. Recall that one of the key advantages of the multilevel design template is a procedure for limiting complex design space exploration to include only areas likely to contain satisficing design solutions. The BRP example problem adds value beyond the cantilever beam example problem (Chapter 4) because of it is more complex in nature. However, with 13 design variables, the BRP example problem can be solved directly without the use of the multilevel design template. BRP design is included in this thesis to illustrate the benefits of the multilevel design template when applied to more complex design problems.

5.2 BLAST RESISTANT PANEL DESIGN PROCESS AND SOLUTION

In the following section, the design of a BRP is presented. First, the BRP design process, as it relates to the multilevel design template, is discussed. Then, the details of the problem formulation, design process, and solutions are examined. The successful implementation of the multilevel design template in the design of a BRP builds confidence in the validation of the multilevel design template.

5.2.1 Multilevel Design Template Particularized for Blast Resistant Panel Design

In the following section, a generic template for multilevel robust design is particularized for the design of a BRP. Recall from Chapter 3 the generic design template for multilevel robust design represented in word form, organized under the headings *Given*, *Define*, *Map*, *Find*, *Satisfy*, *Minimize*.

The domain-general multilevel design template, shown in Figure 3.10, begins with the collection and modeling of design parameters including: design variables, design variable bounds, design constraints, design goals, and design preferences. Design parameters are specified at each level of model complexity. The design parameters are given in the problem statement or determined by an experienced engineer. Following the specification of design parameters, multilevel models of the multilevel design problem are developed at each level of model complexity. Multilevel models are used to predict the response or performance of the design at each level of model complexity.

The generic multilevel design template is particularized for the BRP example problem and is shown in Figure 3.15. At the beginning of the design process, the design parameters (including design variables, bounds on design variables, design constraints, design goals, and design preferences) are given in the problem statement or determined by the designer. After the design parameters are defined, multilevel models of the BRP are developed at each level of model complexity. For the BRP example problem, selection of the multilevel models is based on the levels of complexity in the design problem. The multilevel models defined for the BRP example problem include a BRP consisting of a single solid panel, a BRP consisting of three solid panels, and a BRP with three panels with a honeycomb core. The BRP example problem could be described using more levels of complexity. However, for this example problem, three levels of design complexity are sufficient to capture the behavior of the design. Each multilevel model is linked with mapping functions that map material properties and uncertainty models at each level interface. The deflection and mass of the panel are measured at the *Design Performance* or *Response* section of the design template.

5.2.2 Blast Resistant Panel Design Process

In the following section, the multilevel BRP design process is presented. Design information is organized similar to the information flow of the multilevel design template in word form (Figure 3.7) under the headings, *Given*, *Define*, *Map*, *Find*, *Satisfy*, *Minimize*. The inductive multilevel BRP design solution is given in Section 5.3.3.

Given

The following information provides the underlying assumptions of the BRP design problem.

<u>A Feasible Alternative</u>

BRP design in this thesis is based on a three-layer blast panel with two solid layers surrounding a honeycomb core. It is assumed that the front face sheet receives the initial pressure loading from the blast. The geometry of the core layer is designed to dissipate a majority of the impulse energy in crushing. The back face sheet provides additional protection from the blast as well as a means to confine the core collapse and absorb energy in stretching. A square honeycomb structure is chosen for the cell shape of the core material because the square shape resists stretching of the back face sheet more so than triangular-, hexagonal- and chiral-shaped cores.

<u>Assumptions</u>

The loading conditions and constraints are the same for BRP performance analysis at each level. The impulse load experimented by the BRP is modeled as a uniform pressure wave with a peak pressure of p_0 = 25 MPa and characteristic loading time $t_0 = 10^{-4}$ sec. Equations for BRP loading conditions are shown below (Hutchinson and Xue 2005).

$$p = p_0 e^{-t/t_0} \tag{5.1}$$

$$I_0 = \int p\mathrm{d}t = p_0 t_0 \tag{5.2}$$

The impulse load acts perpendicular to the surface of the BRP and is uniformly distributed over the area of the plate. For deflection calculations, the plate is assumed to be fully clamped at both ends, of width $L/2$, and of infinite extent in the y-direction (Hutchinson and Xue 2005).

For the BRP design material properties are assumed to have an elastic, perfectly-plastic stress-strain relationship, and the material is assumed to be defined by independent yield strength and density variables. The bounds for material property design variables are determined from the ranges of properties for engineering metals. The uncertainty in material design variables is 5% of the range of each material property. The geometric design variables are used to define the height of each layer as well as the cell spacing and cell wall width of the square honeycomb core. Geometric design variables are assumed to have no associated uncertainty. There are two noise factors pertaining to the air blast received by the panel, peak pressure of the incoming pulse and the characteristic time of the pulse. Variation in noise factors is modeled at 15% of the mean value ($\Delta p_0 = 15\% \times 25MPa$, $\Delta t_0 = 15\% \times 10^{-4}\,\mathrm{sec}$).

Parameters

Parameters for BRP design include system variables, constraints, and goals. Design variables are summarized in Table 5.3. BRP design goals include minimizing back face sheet deflection, minimizing mass / area, and maximizing HD-EMI with respect to previously stated performance goals. BRP performance constraints are maximum deflection (δ_{max}) of back face sheet must be less than or equal to 10 % of panel length and maximum mass / area (M) must be less than or equal to 150 kg/m^2. Additional BRP design constraints are discussed later in this section.

Table 5.3 – BRP design variables

Certain Design Variables			
	Lower Bound	Upper Bound	Units
h_f	0.001	0.025	m
h_b	0.001	0.025	m
H	0.001	0.020	m
h_c	0.0001	0.010	m
B	0.005	0.050	m
Uncertain Design Variables			
	Lower Bound	Upper Bound	Units
ρ_f	2000	10000	kg/m^3
ρ_b	2000	10000	kg/m^3
ρ_c	2000	10000	kg/m^5
$\sigma_{Y,f}$	100	1100	MPa
$\sigma_{Y,b}$	100	1100	MPa
$\sigma_{Y,c}$	100	1100	MPa

Define

In the following section design levels, feasible design space, and performance metrics are defined for the BRP multilevel design problem.

Design Levels

Design levels represent the amount of design simplicity or complexity that is considered in reaching a design solution. The BRP design problem is divided into three levels of model complexity. In the following section, a cDSP, performance modeling equations, and modeling techniques are presented for each design level. Recall that when making design decision at a single level, a cDSP is employed.

Level 1

In the following section, the performance of a BRP modeled as one solid layer analyzed (see Figure 5.8). At Level 1 BRP design only 3 design variables are considered (σ_y, ρ, h_{total}).

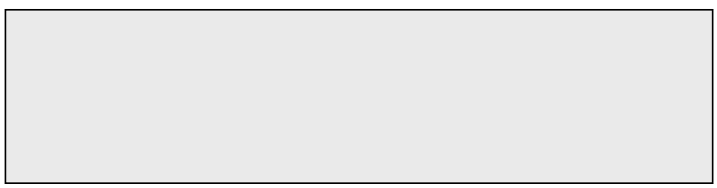


Figure 5.7 – Multilevel model 1: One solid panel

A cDSP for Level 1 BRP design is presented in Table 5.4.

Table 5.4 – BRP Level 1 cDSP

Word Formulation	**Mathematical Formulation**
Given	*Given*
BRP with a single, solid layer	BRP length: L = 1 m
Uniform pressure impulse	Impulse load model: $p = p_0 e^{-t/t_0}$ $I_0 = \int p\mathrm{d}t = p_0 t_0$ $p_0 = 25\,\mathrm{MPa},\ t_0 = 10^{-4}\,\mathrm{sec}$
Loading uncertainty models	$\Delta p_0 = 0.15\mu_{p_0},\ \Delta t_0 = 0.15\mu_{t_0}$
Material property uncertainty models	$\Delta\sigma_y = 0.05\left(\sigma_{y,ub} - \sigma_{y,lb}\right),\ \Delta\rho = 0.05\left(\rho_{ub} - \rho_{lb}\right)$
BRP performance model	$\delta = f\left(h, \sigma_y, \rho\right),\ M = f\left(h, \rho\right)$

Table 5.4 (continued) – BRP Level 1 cDSP

Find	*Find*
BRP design variables BRP deviation variables	Dimensions of BRP: (h) Material properties of each layer of BRP: (ρ, σ_y) d_i^+, d_i^-
Satisfy	*Satisfy*
Constraints Maximum allowable deflection Maximum allowable mass / area	*Constraints* $\delta + \Delta\delta \leq 10$ cm $M + \Delta M \leq 150$ kg/m^2
Bounds density yield strength panel thickness deviation variables	*Bounds* 2000 kg/m$^3 \leq \rho \pm \Delta\rho \leq$ 10000 kg/m^3 100 MPa $\leq \sigma_y \pm \Delta\sigma_y \leq$ 1100 MPa 7 mm $\leq h \leq$ 100 mm $d_i^+, d_i^- \geq 0$ $d_i^+ \cdot d_i^- = 0$
Goals Minimize deflection (δ) Minimize mass/area (M) Maximize HD-EMI$^{\delta}$ Maximize HD-EMI^{M}	*Goals* $d_1^- = 1 - G_1 / A_1(x)$ $W_1 = 0.25$ $d_2^- = 1 - G_2 / A_2(x)$ $W_2 = 0.25$ $d_3^- = 1 - A_3(x) / G_3$ $W_3 = 0.25$ $d_4^- = 1 - A_4(x) / G_4$ $W_4 = 0.25$
Minimize	*Minimize*
Deviation from target	$Z = W_1 d_1^- + W_2 d_2^- + W_3 d_3^- + W_4 d_4^-$

Equations used in predicting BRP performance at Level 1 are given in the following paragraphs. The three-stage deformation process developed by Fleck and Deshpande is implemented in performance analysis of a BRP with one solid layer. The three-stage deformation process is modified to reflect the performance of a single panel under impulse loading. The following deflection equations are adapted from the work of Hutchinson and Xue, 2005. Following the three stage deformation theory, the impulse of the blast is received by the front face sheet and momentum is transferred in stage one (Fleck and Deshpande 2004). The equation for kinetic energy per unit area at the end of stage one is given in Equation 5.3. In stage two, the deflection equation has been modified to reflect no core crushing and a BRP consisting of a single panel. The equation for the amount of kinetic energy per unit area at the end of stage two is shown in Equation 5.4.

$$KE_I = \frac{2 p_0^2 t_0^2}{\rho h} \tag{5.3}$$

$$KE_{II} = \frac{2I_0^2}{\rho h} \tag{5.4}$$

In stage three, the remaining kinetic energy must be dissipated through bending and stretching of the panel. The equation for deflection is derived by equating the remaining kinetic energy per unit area to the plastic work per unit area dissipated through bending and stretching. The average plastic work per unit area dissipated in stage three is estimated by summing the dissipation from bending and stretching, following the work of Hutchinson and Xue, 2005. The equation for this estimate is shown in Equation 5.5. The equation for deflection is shown in Equation 5.6. Details regarding the calculation of the deflection of a BRP at Level 1 are given in Appendix C.

$$W_{III}^{p} = \frac{2}{3}\left[\sigma_y h\right]\left(\frac{\delta}{L}\right)^2 + \sigma_y h \frac{h}{L}\left(\frac{\delta}{L}\right) \tag{5.5}$$

$$\delta = \frac{-\sigma_y h^2 \pm \sqrt{\sigma_y^2 h^4 + \frac{16}{3}\frac{L^2\left[\sigma_y h\right]\left(p_0^2 t_0^2\right)}{\rho h}}}{\frac{4}{3}\left[\sigma_y h\right]} \tag{5.6}$$

Equations for the variation in deflection are also needed in order to determine the sensitivity of the panel to variation in noise factors and uncertain design variables. The derived equation for the variation in deflection is shown in Equation 5.7. Extensive details regarding the calculation of the variance of deflection of a BRP are given in Appendix C.

$$\Delta\delta = \left|\frac{\partial\delta}{\partial\sigma_y}\right|\Delta\sigma_y + \left|\frac{\partial\delta}{\partial\rho}\right|\Delta\rho + \left|\frac{\partial\delta}{\partial p_0}\right|\Delta p_0 + \left|\frac{\partial\delta}{\partial t_0}\right|\Delta t_0 \tag{5.7}$$

Performance and design variable constraints for a BRP at Level 1 are given in the following paragraphs. For a BRP modeled as three solid layers, the derivation of the maximum mass per unit area constraint is shown in Equation 5.8, and the derivation of the maximum deflection constraint is shown in Equation 5.9.

$$\begin{aligned}
&M - 150kg/m^2 \leq 0 \\
&M = f\left(\rho, \sigma_y, h_{total}\right) \\
&g_M = f\left(\rho, \sigma_y, h_{total}\right) - 150kg/m^2 \\
&\Delta g_m = \left|\frac{\partial g_m}{\partial \rho_f}\right| \Delta \rho_f \\
&g_M + \Delta g_M \leq 0
\end{aligned} \tag{5.8}$$

$$\begin{aligned}
&\delta - 0.1L \leq 0 \\
&\delta = f\left(\rho, \sigma_y, h_{total}\right) \\
&g_\delta = f\left(\rho, \sigma_y, h_{total}\right) - (0.1)L \\
&\Delta g_\delta = \left|\frac{\partial g_m}{\partial \rho}\right| \Delta \rho + \left|\frac{\partial g_m}{\partial \sigma_y}\right| \Delta \sigma_y \\
&g_\delta + \Delta g_\delta \leq 0
\end{aligned} \tag{5.9}$$

Constraints to keep the material property design variables within the specified bounds in spite of the assumed uncertainty in these variables are imposed. A generic form of these constraints is shown in Equation 5.10.

$$\begin{aligned}
&(\text{lower bound}) - \left(x - \tfrac{1}{2}\Delta x\right) \leq 0 \\
&\left(x + \tfrac{1}{2}\Delta x\right) - (\text{upper bound}) \leq 0 \\
&\forall x = \left\{\rho, \sigma_y, h_{total}\right\}
\end{aligned} \tag{5.10}$$

Solutions for BRP design at each level are found by implementing a cDSP using the `fmincon()` function in MATLAB (MATLAB 2004). A graphical user interface (GUI) called the Blast Resistant Panel Design Studio is developed to gather user input data for BRP design solution analysis. Developing BRP analysis models and the BRP Design Studio is a collaboration with Georgia Tech student, Stephanie Thompson. The BRP GUI is shown in Figure 5.8.

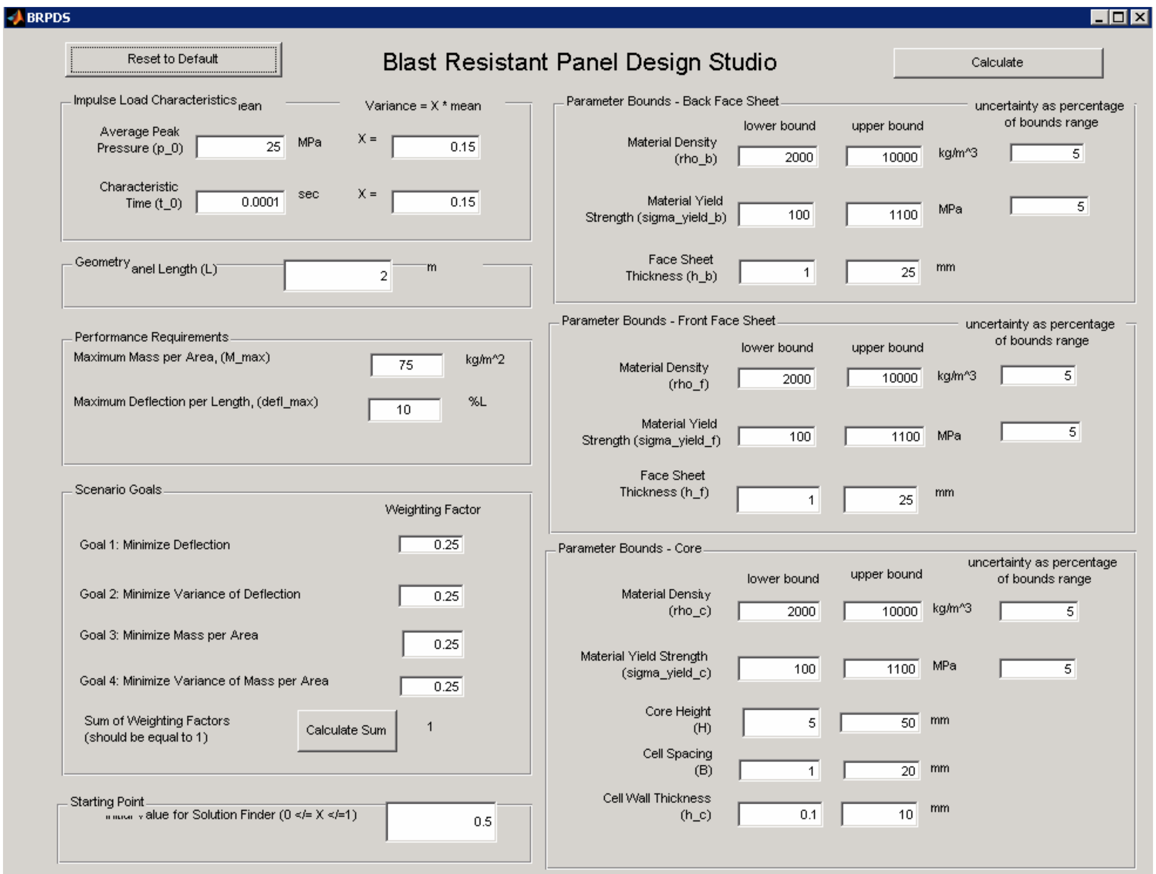


Figure 5.8 – BRP Design Studio, a graphical user interface implemented in MATLAB

Level 2

In the following section, the performance of a BRP consisting of three solid layers is analyzed (see Figure 5.10). The BRP analyzed in the following section is different from the BRP analyzed in the previous section in that model complexity is significantly increased. In the previous BRP analysis model, 3 design variables are used to describe a design solution. The BRP performance model analyzed in the following section considers 9 design variables (σ_y, σ_y, σ_y, ρ_f, ρ_b, ρ_c, h_f, h_b, H).

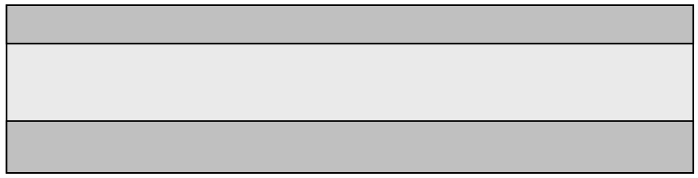


Figure 5.9 – Multilevel model 2: Three solid panels

A cDSP for Level 2 BRP design is presented in Table 5.5.

Table 5.5 – BRP Level 2 cDSP

Word Formulation	**Mathematical Formulation**
Given	*Given*
BRP with three solid layers	BRP length: L = 1 m
Uniform pressure impulse	Impulse load model: $p = p_0 e^{-t/t_0}$ $I_0 = \int p\mathrm{d}t = p_0 t_0$ $p_0 = 25\,\mathrm{MPa},\ t_0 = 10^{-4}\,\mathrm{sec}$
Loading uncertainty models	$\Delta p_0 = 0.15\mu_{p_0},\ \Delta t_0 = 0.15\mu_{t_0}$
Material property uncertainty models	$\Delta\sigma_y = 0.05(\sigma_{y,ub} - \sigma_{y,lb}),\ \Delta\rho = 0.05(\rho_{ub} - \rho_{lb})$
BRP performance model	$\delta = f(h_f, h_b, H, \sigma_{y,i}, \rho_i)$ $M = f(h_f, h_b, H, \rho_i)$
Find	*Find*
BRP design variables BRP deviation variables	Dimensions of BRP: (h_f, h_b, H) Material properties of each layer of BRP: (ρ_i , σ_{yi}) d_i^+, d_i^-
Satisfy	*Satisfy*
Constraints Maximum allowable deflection Maximum allowable mass / area	*Constraints* $\delta + \Delta\delta \leq 10$ cm $M + \Delta M \leq 150$ kg/m^2

Table 5.5 (continued) – BRP Level 2 cDSP

Bounds density yield strength front face sheet thickness back face sheet thickness core height deviation variables	*Bounds* 2000 kg/m$^3 \leq \rho \pm \Delta\rho \leq$ 10000 kg/m^3 100 MPa $\leq \sigma_{\mathrm{y}} \pm \Delta\sigma_{\mathrm{y}} \leq$ 1100 MPa 1 mm $\leq h_f \leq$ 25 mm 1 mm $\leq h_b \leq$ 25 mm 5mm $\leq H \leq$ 50 mm $d_i^+, d_i^- \geq 0$ $d_i^+ \cdot d_i^- = 0$
Goals Minimize deflection (δ) Minimize mass/area (M) Maximize HD-EMI$^{\delta}$ Maximize HD-EMI^{M}	*Goals* $d_1^- = 1 - G_1 / A_1(x)$ $W_1 = 0.25$ $d_2^- = 1 - G_2 / A_2(x)$ $W_2 = 0.25$ $d_3^- = 1 - A_3(x) / G_3$ $W_3 = 0.25$ $d_4^- = 1 - A_4(x) / G_4$ $W_4 = 0.25$
Minimize	*Minimize*
Deviation from target	$Z = W_1d_1^- + W_2d_2^- + W_3d_3^- + W_4d_4^-$

The three-stage deformation process developed by Fleck and Deshpande is implemented in performance analysis of a BRP with three solid layers. The three-stage deformation process is modified slightly in Stage II, core crushing. Since a BRP consisting of three solid layers does not experience core crushing, equations reflecting this portion of BRP deformation have been modified to reflect the non-crushing nature of a BRP with three solid layers. These modifications are reflected in the equations that follow. The deflection equations developed for a BRP with three solid layers are adapted from the work of Hutchinson and Xue, 2005. Similar assumptions regarding loading conditions and material properties discussed in BRP modeling at Level 1 are applied to BRP modeling at Level 2.

The equations for deflection of the back face sheet developed by Hutchinson and Xue are extended to allow for independent layer height, and independent material in each layer, and the modification of the core layer to resemble a solid panel. Following the three stage deformation theory, the impulse of the blast is received by the front face sheet and momentum is transferred in stage one (Fleck and Deshpande 2004). The equation for kinetic energy per unit area at the end of stage one is given in Equation 5.11. In stage two, the deflection equation has been modified to reflect no core crushing. The equation for the amount of kinetic energy per unit area at the end of stage two is shown in Equation 5.12.

$$KE_{I} = \frac{2 p_0^2 t_0^2}{\rho_f h_f} \tag{5.11}$$

$$KE_{II} = \frac{2 I_0^2}{\rho_f h_f + \rho_c H + \rho_b h_b} \tag{5.12}$$

In stage three, the remaining kinetic energy must be dissipated through bending and stretching of the back face sheet. The equation for deflection is derived by equating the remaining kinetic energy per unit area to the plastic work per unit area dissipated through bending and stretching. The average plastic work per unit area dissipated in stage three is estimated by summing the dissipation from bending and stretching, following the work of Hutchinson and Xue, 2005. The equation for this estimate is shown in Equation 5.13. The equation for deflection is shown in Equation 5.14. Details regarding the calculation of the deflection of BRPs are given in Appendix C.

$$W_{III}^{p} = \frac{2}{3}\left[\sigma_{y,f} h_f + \sigma_{y,c} H + \sigma_{y,b} h_b\right]\left(\frac{\delta}{L}\right)^2 + \sigma_{y,b} h_b \frac{H}{L}\left(\frac{\delta}{L}\right) \tag{5.13}$$

$$\delta = \frac{-\sigma_{y.b} h_b H \pm \sqrt{\sigma_{y,b}^2 h_b^2 H^2 + \frac{16}{3}\frac{L^2\left[\sigma_{y,f} h_f + \sigma_{y,c} H + \sigma_{y,b} h_b\right]\left(p_0^2 t_0^2\right)}{\rho_f h_f + \rho_c H + \rho_b h_b}}}{\frac{4}{3}\left[\sigma_{y,f} h_f + \sigma_{y,c} H + \sigma_{y,b} h_b\right]} \tag{5.14}$$

Equations for the variation in deflection are also needed in order to determine the sensitivity of the panel to variation in noise factors and uncertain design variables. The derived equation for the variation in deflection is shown in Equation 5.15. Extensive details regarding the calculation of the variance of deflection of a BRP are given in Appendix C.

$$\begin{aligned}
\Delta\delta = &\left|\frac{\partial\delta}{\partial\sigma_{y,b}}\right|\Delta\sigma_{y,b} + \left|\frac{\partial\delta}{\partial\sigma_{y,c}}\right|\Delta\sigma_{y,c} + \left|\frac{\partial\delta}{\partial\sigma_{y,f}}\right|\Delta\sigma_{y,f} + \ldots \\
&\left|\frac{\partial\delta}{\partial\rho_b}\right|\Delta\rho_b + \left|\frac{\partial\delta}{\partial\rho_c}\right|\Delta\rho_c + \left|\frac{\partial\delta}{\partial\rho_f}\right|\Delta\rho_f + \ldots \\
&\left|\frac{\partial\delta}{\partial p_0}\right|\Delta p_0 + \left|\frac{\partial\delta}{\partial t_0}\right|\Delta t_0
\end{aligned} \tag{5.15}$$

In order to transfer design information from Level 2 to Level 1, material mapping functions are developed. The material properties of a BRP modeled as three solid layers (Level 2) are mapped to a BRP defined by one solid layer (Level 1) by averaging the material properties of a BRP at Level 2, accounting for differences in panel height. A description of how material properties mapped from Level 2 to Level 1 is given in Equation 5.16 and Equation 5.17. The height of a BRP modeled as a single panel is the sum of panel height for each panel in a BRP modeled using three solid layers. This relationship is given in Equation 5.18.

$$\begin{aligned}
&\rho_{level1} = \rho_{b,level2}\left(h_{b,level2} / h_{total,level2}\right) + \ldots \\
&\rho_{c.level2}\left(H_{level2} / h_{total,level2}\right) + \rho_{f,level2}\left(h_{f,level2} / h_{total,level2}\right)
\end{aligned} \tag{5.16}$$

$$\begin{aligned}
&\sigma_{y,level1} = \sigma_{y,b,level2}\left(h_{b,level2} / h_{total,level2}\right) + \ldots \\
&\sigma_{y,c,level2}\left(H_{level2} / h_{total,level2}\right) + \ldots \\
&\sigma_{y,f,level2}\left(h_{f,level2} / h_{total,level2}\right)
\end{aligned} \tag{5.17}$$

$$h_{total} = h_b + H + h_f \tag{5.18}$$

For a BRP modeled as three solid layers, the derivation of a maximum mass per unit area constraint is shown in Equation 5.19, and a derivation of the maximum deflection constraint is shown in Equation 5.20.

$$\begin{aligned}
&M - 150kg/m^2 \leq 0 \\
&M = f\left(\rho_f, \rho_{c,effective}, \rho_b, H, h_f, h_b\right) \\
&g_M = f\left(\rho_f, \rho_{c,effective}, \rho_b, H, h_f, h_b\right) - 150kg/m^2 \\
&\Delta g_m = \left|\frac{\partial g_m}{\partial \rho_f}\right| \Delta\rho_f + \left|\frac{\partial g_m}{\partial \rho_{c,effective}}\right| \Delta\rho_{c,effective} + \left|\frac{\partial g_m}{\partial \rho_b}\right| \Delta\rho_b \\
&g_M + \Delta g_M \leq 0
\end{aligned} \tag{5.19}$$

$$\begin{aligned}
&\delta - 0.1L \leq 0 \\
&\delta = f\left(\rho_f, \rho_{c,effective}, \rho_b, \sigma_{y,f}, \sigma_{y,c,effective}, \sigma_{y,b}, H, h_f, h_b\right) \\
&g_\delta = f\left(\rho_f, \rho_{c,effective}, \rho_b, \sigma_{y,f}, \sigma_{y,c,effective}, \sigma_{y,b}, H, h_f, h_b\right) - (0.1)L \\
&\Delta g_\delta = \left|\frac{\partial g_m}{\partial \rho_f}\right| \Delta\rho_f + \left|\frac{\partial g_m}{\partial \rho_{c,effective}}\right| \Delta\rho_{c,effective} + \left|\frac{\partial g_m}{\partial \rho_b}\right| \Delta\rho_b + \ldots \\
&\left|\frac{\partial g_m}{\partial \sigma_{y,f}}\right| \Delta\sigma_{y,f} + \left|\frac{\partial g_m}{\partial \sigma_{y,c,effective}}\right| \Delta\sigma_{y,f} + \left|\frac{\partial g_m}{\partial \sigma_{y,c,effective}}\right| \Delta\sigma_{y,c,effective} \\
&g_\delta + \Delta g_\delta \leq 0
\end{aligned} \tag{5.20}$$

Finally, constraints to keep the material property design variables within the specified bounds in spite of the assumed uncertainty in these variables are imposed. A generic form of these constraints is shown in Equation 5.21. There are two such constraints for each of the six uncertain design variables. Similarly to the previous section, it is assumed that material property uncertain is modeled as 5% of the corresponding material bound.

$$
\begin{aligned}
&(\text{lower bound}) - \left(x - \tfrac{1}{2}\Delta x\right) \le 0 \\
&\left(x + \tfrac{1}{2}\Delta x\right) - (\text{upper bound}) \le 0 \\
&\forall x = \left\{\rho_b, \rho_c, \rho_f, \sigma_{Y,b}, \sigma_{Y,c}, \sigma_{Y,f}\right\}
\end{aligned}
\tag{5.21}
$$

Level 3

In the following section, a BRP consisting of three layers and a honeycomb core is analyzed (see Figure 5.10). In this section, BRP performance modeling is at the highest level of model complexity examined in this thesis.

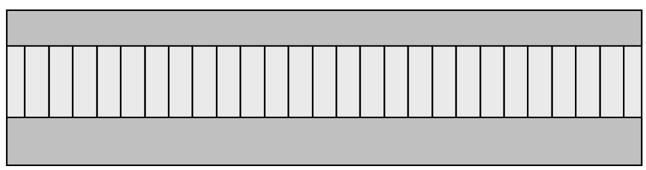


Figure 5.10 – Multilevel model 3: Solid panels surrounding honeycomb core

A cDSP for Level 3 BRP design is presented in Table 5.6.

Table 5.6 – BRP Level 3 cDSP

Word Formulation	**Mathematical Formulation**
Given	*Given*
BRP with two solid layers surrounding a honeycomb core with a square topology	BRP length: L = 1 m
Uniform pressure impulse	Impulse load model: $p = p_0 e^{-t/t_0}$ $I_0 = \int p\mathrm{d}t = p_0 t_0$ $p_0 = 25\,\text{MPa},\ t_0 = 10^{-4}\,\text{sec}$
Loading uncertainty models	$\Delta p_0 = 0.15\mu_{p_0},\ \Delta t_0 = 0.15\mu_{t_0}$
Material property uncertainty models	$\Delta\sigma_y = 0.05\left(\sigma_{y,ub} - \sigma_{y,lb}\right),\ \Delta\rho = 0.05\left(\rho_{ub} - \rho_{lb}\right)$
BRP performance model	$\delta = f\left(h_f, h_b, H, h_c, B, \sigma_y, \rho\right)$ $M = f\left(h_f, h_b, H, h_c, B, \rho\right)$

Table 5.6 (continued) – BRP Level 3 cDSP

Find	*Find*
BRP design variables	Dimensions of BRP:
	(h_f, h_b, H, h_c, B) Material properties of each layer of BRP: (ρ_i , σ_{yi}) for $i = f, b, c$
BRP deviation variables	d_i^+, d_i^-
Satisfy	*Satisfy*
Constraints Maximum allowable deflection Maximum allowable mass / area Relative density of core constraint Shear-off constraints	*Constraints* $\delta + \Delta\delta \leq 10$ cm $M + \Delta M \leq 150$ kg/m^2 $R_c \geq 0.07$ $g_{SH1} = f\left(h_f, \rho_f, \sigma_{y,f}, p_0, t_0\right)$ $g_{SH2} = f\left(h_f, H, h_c, B, \rho\right)$
Bounds density yield strength front face sheet thickness back face sheet thickness core height cell wall thickness cell spacing deviation variables	*Bounds* 2000 kg/m$^3 \leq \rho \pm \Delta\rho \leq$ 10000 kg/m^3 100 MPa $\leq \sigma_y \pm \Delta\sigma_y \leq$ 1100 MPa 1 mm $\leq h_f \leq$ 25 mm 1 mm $\leq h_b \leq$ 25 mm 5mm $\leq H \leq$ 50 mm 0.1 mm $\leq h_c \leq$ 10 mm 1 mm $\leq B \leq$ 20 mm $d_i^+, d_i^- \geq 0$ $d_i^+ \cdot d_i^- = 0$
Goals Minimize deflection (δ) Minimize mass/area (M) Maximize HD-EMI$^\delta$ Maximize HD-EMI^M	*Goals* $d_1^- = 1 - G_1 / A_1(x)$ $W_1 = 0.25$ $d_2^- = 1 - G_2 / A_2(x)$ $W_2 = 0.25$ $d_3^- = 1 - A_3(x) / G_3$ $W_3 = 0.25$ $d_4^- = 1 - A_4(x) / G_4$ $W_4 = 0.25$
Minimize	*Minimize*
Deviation from target	$Z = W_1d_1^- + W_2d_2^- + W_3d_3^- + W_4d_4^-$

The deformation of sandwich plates under impulse loading is divided into three time periods according to the three-stage deformation theory developed by Fleck and Deshpande, 2004. Fleck and Deshpande analyze the stages of deformation, and propose equations relating impulse loading to deformation. In subsequent work, Hutchinson and Xue adapted the three-stage deformation theory proposed by Fleck and Deshpande and applied it to the optimization of blast resistant panels (Hutchinson and Xue 2005).

The equations for deflection of the back face sheet developed by Hutchinson and Xue (2005) are extended to allow for independent layer heights, and independent materials in each layer. Following the three stage deformation theory, the impulse of the blast is received by the front face sheet and momentum is transferred in stage one (Fleck and Deshpande 2004). The equation for kinetic energy per unit area at the end of stage one is given in Equation 5.22. In stage two, some of the kinetic energy is dissipated through crushing of the core layer. The equation for the amount of kinetic energy per unit area at the end of stage two is shown in Equation 5.23. The crushing strain is used to determine the crushed height of the core layer and is derived by equating the plastic dissipation per unit area in the core to the loss of kinetic energy per unit area in stage two (Hutchinson and Xue 2005). The crushing strain in is shown in Equation 5.24.

$$KE_I = \frac{2 p_0^2 t_0^2}{\rho_f h_f} \tag{5.22}$$

$$KE_{II} = \frac{2 p_0^2 t_0^2}{\rho_f h_f + \rho_c R_c H + \rho_b h_b} \tag{5.23}$$

$$\bar{\varepsilon}_c = \frac{2 p_0^2 t_0^2 \left(\rho_b h_b + \rho_c R_c H\right)}{\lambda_c R_c \sigma_{y,c} H \rho_f h_f \left(\rho_f h_f + \rho_c R_c H + \rho_b h_b\right)} \tag{5.24}$$

In stage three, the remaining kinetic energy must be dissipated through bending and stretching of the back face sheet. The equation for deflection is derived by equating the remaining kinetic energy per unit area to the plastic work per unit area dissipated through bending and stretching. The average plastic work per unit area dissipated in stage three is estimated by summing the dissipation from bending and stretching, following the work of Hutchinson and Xue, 2005. The equation for this estimate is shown in Equation 5.25. The equation for deflection is shown in Equation 5.26. Details regarding the calculation of the deflection of BRPs are given in Appendix C.

$$W_{III}^{P} = \frac{2}{3}\left[\sigma_{Y,f} h_f + \sigma_{Y,c} R_c H \lambda_S + \sigma_{Y,b} h_b\right]\left(\frac{\delta}{L}\right)^2 + 4\sigma_{Y,b} h_b \frac{\overline{H}}{L}\left(\frac{\delta}{L}\right) \tag{5.25}$$

$$\delta = \frac{-3\left(\sigma_{y,b} h_b \overline{H}\right) \pm \sqrt{9\sigma_{y,b}^2 h_b^2 \overline{H}^2 + \left(\frac{3I_0^2 L^2\left[\sigma_{y,f} h_f + \sigma_{y,c} R_c H \lambda_S + \sigma_{y,b} h_b\right]}{\left(\rho_f h_f + \rho_c R_c H + \rho_b h_b\right)}\right)}}{\left[\sigma_{y,f} h_f + \sigma_{y,c} R_c H \lambda_S + \sigma_{y,b} h_b\right]} \tag{5.26}$$

where $\overline{H} = H\left(1-\overline{\varepsilon}_c\right) = \left(H - \frac{2I_0^2\left(\rho_b h_b + \rho_c R_c H\right)}{\lambda_c R_c \sigma_{y,c} \rho_f h_f \left(\rho_f h_f + \rho_b h_b + \rho_c R_c H\right)}\right)$

Equations for the variation in deflection are also needed in order to determine the sensitivity of the panel to variation in noise factors and uncertain design variables. The derived equation for the variation in deflection is shown in Equation 5.27. Extensive details regarding the calculation of the variance of deflection of a BRP are given in Appendix C.

$$\begin{aligned}\Delta\delta = &\left|\frac{\partial\delta}{\partial\sigma_{y,b}}\right|\Delta\sigma_{y,b} + \left|\frac{\partial\delta}{\partial\sigma_{y,c}}\right|\Delta\sigma_{y,c} + \left|\frac{\partial\delta}{\partial\sigma_{y,f}}\right|\Delta\sigma_{y,f} + \ldots \\ &\left|\frac{\partial\delta}{\partial\rho_b}\right|\Delta\rho_b + \left|\frac{\partial\delta}{\partial\rho_c}\right|\Delta\rho_c + \left|\frac{\partial\delta}{\partial\rho_f}\right|\Delta\rho_f + \ldots \\ &\left|\frac{\partial\delta}{\partial p_0}\right|\Delta p_0 + \left|\frac{\partial\delta}{\partial t_0}\right|\Delta t_0\end{aligned} \tag{5.27}$$

In order to pass design information from Level 3 to Level 2, material mapping functions are created. The core layer of a BRP at Level 2 is modeled as a solid layer with effective material properties. Effective material properties (ρ_c, $\sigma_{y,c}$) for the core at Level 2 are developed based on the material properties of the core layer and the relative density of the core layer at Level 3. The effective density of the core layer of a BRP at Level 2 is the relative density (R_c) of the core at Level 3 multiplied by the density of the core (ρ_c) at Level 3. This relationship is given in Equation 5.28.

$$\rho_{c,level2} = \rho_{c,level3} R_{c,level3} \tag{5.28}$$

According to the work of Wang and McDowell for periodic metal honeycombs under uniaxial loading, the effective yield strength is approximated as the relative density of the honeycomb multiplied by the yield strength of the honeycomb base material (Wang and McDowell 2005). The effective yield strength for the core layer in a BRP modeled as three solid layers is given in Equation 5.29.

$$\sigma_{y,c,level2} = \sigma_{y,c,level3} R_{c,level3} \tag{5.29}$$

The BRP design is constrained to limit mass and deflection, prohibit failure, and maintain the bounds of the design space. All the constraints that are a function of uncertain control or noise factors are imposed as “robust constraints”. That is, the variation of these quantities as a result of variation in the uncertain factors is included when evaluating the constraints. An explanation of system constraints and their derivation follows.

A maximum mass per unit area constraint is imposed to limit the panel to 150 kg/m^2. In addition, a maximum allowable deflection constraint is defined as 10% of the panel length. Although one of the performance goals of the design is to minimize deflection, this constraint is imposed to keep the deflection small enough so that assumptions in the response model will not be violated. The derivation of the maximum mass per unit area constraint is shown in Equation 5.30, and the derivation of the maximum deflection constraint is shown in Equation 5.31.

$$
\begin{aligned}
& M - 150kg/m^2 \leq 0 \\
& M = f\left(\rho_f, \rho_f, \rho_f, B, H, h_c, h_f, h_b\right) \\
& g_M = f\left(\rho_f, \rho_f, \rho_f, B, H, h_c, h_f, h_b\right) - 150kg/m^2 \\
& \Delta g_M = \left|\frac{\partial g_M}{\partial \rho_f}\right| \cdot \Delta\rho_f + \left|\frac{\partial g_M}{\partial \rho_b}\right| \cdot \Delta\rho_b + \left|\frac{\partial g_M}{\partial \rho_c}\right| \cdot \Delta\rho_c \\
& g_M + \Delta g_M \leq 0
\end{aligned}
\tag{5.30}
$$

$$
\begin{aligned}
& \delta - 0.1L \leq 0 \\
& \delta = f\left(p_0, t_0, \sigma_{Y,b}, \rho_b, \sigma_{Y,c}, \rho_c, \sigma_{Y,f}, \rho_f, B, H, h_c, h_f, L\right) \\
& g_\delta = f\left(p_0, t_0, \sigma_{Y,b}, \rho_b, \sigma_{Y,c}, \rho_c, \sigma_{Y,f}, \rho_f, B, H, h_c, h_f, L\right) - (0.1)L \\
& \Delta g_\delta = \left|\frac{\partial g_\delta}{\partial p_0}\right| \cdot \Delta p_0 + \left|\frac{\partial g_\delta}{\partial t_0}\right| \cdot \Delta t_0 + \left|\frac{\partial g_\delta}{\partial \sigma_{Y,b}}\right| \cdot \Delta\sigma_{Y,b} + \left|\frac{\partial g_\delta}{\partial \rho_b}\right| \cdot \Delta\rho_b + \\
& \qquad \left|\frac{\partial g_\delta}{\partial \sigma_{Y,c}}\right| \cdot \Delta\sigma_{Y,c} + \left|\frac{\partial g_\delta}{\partial \rho_c}\right| \cdot \Delta\rho_c + \left|\frac{\partial g_\delta}{\partial \sigma_{Y,f}}\right| \cdot \Delta\sigma_{Y,f} + \left|\frac{\partial g_\delta}{\partial \rho_f}\right| \cdot \Delta\rho_f \\
& g_\delta + \Delta g_\delta \leq 0
\end{aligned}
\tag{5.31}
$$

The relative density (R_c) of the core is the amount of material in the core divided by the total volume of the core. A minimum relative density of the core constraint is imposed to ensure crushing of the core rather than buckling, which would not dissipate as much energy. The relative density is not a function of uncertain factors. The equation for the calculation of the relative density constraint is shown in Equation 5.32.

$$
\begin{aligned}
& R_c = \left(2Bh_c - h_c^2\right) / B^2 \\
& 0.07 - R_c \leq 0
\end{aligned}
\tag{5.32}
$$

Two constraints to avoid shear failure of the front face sheet are imposed. The first criterion prohibits shear of the face sheet at the clamped ends of the plate, and the second criterion prohibits shear of the face sheet at the core webs. These constraints are shown in Equations 5.33 and 5.34, respectively.

$$
\begin{aligned}
& g_{SH1} = \left(2 p_0 t_0 \Big/ h_f \sqrt{\sigma_{Y,f} \rho_f} \right) - \Gamma_{SH}, \text{ where } \Gamma_{SH} = 0.6 \\
& \Delta g_{SH1} = \left| \frac{\partial g_{SH1}}{\partial p_0} \right| \Delta p_0 + \left| \frac{\partial g_{SH1}}{\partial t_0} \right| \Delta t_0 + \left| \frac{\partial g_{SH1}}{\partial \sigma_{Y,f}} \right| \Delta \sigma_{Y,f} + \left| \frac{\partial g_{SH1}}{\partial \rho_f} \right| \Delta \rho_f \\
& g_{SH1} + \Delta g_{SH1} \le 0
\end{aligned}
\tag{5.33}
$$

$$
\begin{aligned}
& g_{SH2} = \frac{\rho_c H R_c}{\rho_f h_f} - \frac{4}{\sqrt{3}} \\
& \Delta g_{SH2} = \left| \frac{\partial g_{SH2}}{\partial \rho_f} \right| \Delta \rho_f + \left| \frac{\partial g_{SH2}}{\partial \rho_c} \right| \Delta \rho_c \\
& g_{SH2} + \Delta g_{SH2} \le 0
\end{aligned}
\tag{5.34}
$$

Finally, constraints to keep the material property design variables within the specified bounds in spite of the assumed uncertainty in these variables are imposed. A generic form of these constraints is shown in Equation 5.35. There are two such constraints for each of the six uncertain design variables. It is assumed that material property uncertain is modeled as 5% of the corresponding material bound.

$$
\begin{aligned}
& (\text{lower bound}) - \left(x - \tfrac{1}{2} \Delta x \right) \le 0 \\
& \left(x + \tfrac{1}{2} \Delta x \right) - (\text{upper bound}) \le 0 \\
& \forall x = \left\{ \rho_b, \rho_c, \rho_f, \sigma_{Y,b}, \sigma_{Y,c}, \sigma_{Y,f} \right\}
\end{aligned}
\tag{5.35}
$$

Feasible Design Space

The feasible design space at each level of design complexity is defined by bounds placed on design variables. In BRP design, bounds on material property design variables are based on properties of all metals. Bounds placed on geometric design variables are determined based on blast panel manufacturability. A summary of the feasible design space for each level of model complexity is given in Table 5.7.

Table 5.7 – Bounds on design variables in BRP design

Design Variable	Lower Bound	Upper Bound
$\sigma_{y,f}$	100 MPa	1100 MPa
$\sigma_{y,c}$	100 MPa	1100 MPa
$\sigma_{y,b}$	100 MPa	1100 MPa
ρ_f	2000 kg/m^3	1000 kg/m^3
ρ_c	2000 kg/m^3	1000 kg/m^3
ρ_b	2000 kg/m^3	1000 kg/m^3
B	1 mm	20 mm
H	5 mm	50 mm
h_c	0.1 mm	10 mm
h_f	1 mm	25 mm
h_b	1 mm	25 mm

Map

Once product performance is modeled at various levels of design complexity, a method for linking the models, both deductively and inductively, is developed. For a generic multilevel design process, mapping functions describing design variables and uncertainty models should be developed at each level-to-level interface. For the BRP example problem, mapping functions are created to describe the relationship of material properties and uncertainty models when traveling from one multilevel model to another. These mapping functions are mathematical descriptions of material and uncertainty information propagation in a multilevel design process. Details regarding developed mapping functions for the BRP example problem are given in the following section.

Material Mapping Functions

A visual representation used to describe material property mappings among the three levels in the BRP design problem is given in the Figure 5.11 and Figure 5.12. The material mappings are applied to all material properties used in BRP performance analysis and include density (ρ) and yield strength (σ_y).

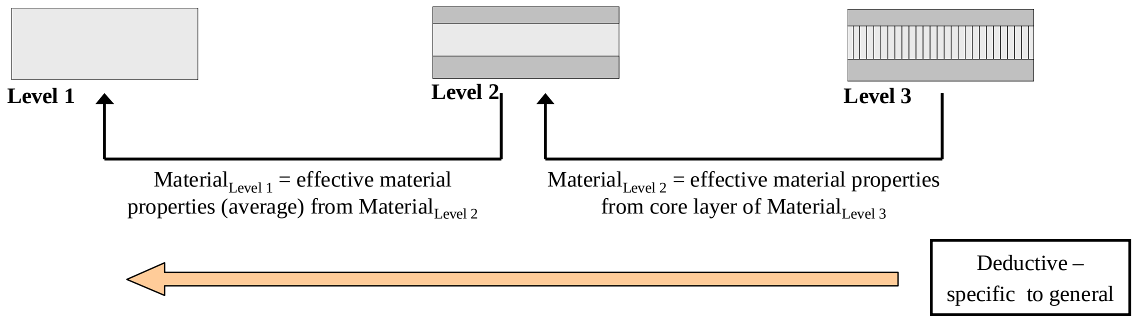


Figure 5.11 – Mapping functions of material properties in BRP design (deductive)

A description of the specific mapping functions follows:

Deductive mapping functions

- *Level 3 to Level 2* – Material properties of the core layer at Level 2 are determined by multiplying the material properties of the core layer at Level 3 by the relative density of the core at Level 3. Recall from Equations 5.28 – 5.29 that this assumption holds for metallic honeycomb cores under uniaxial loading conditions (Wang, et al. 2005).
- *Level 2 to Level 1* – The homogeneous material properties of the single layer BRP at Level 1 are determined by averaging the material properties of the three layers of the BRP in Level 2, as in Equations 5.16 – 5.18. In this case, averaging depends on the material properties as well as the height of the layer, compared to the total height of the panel.

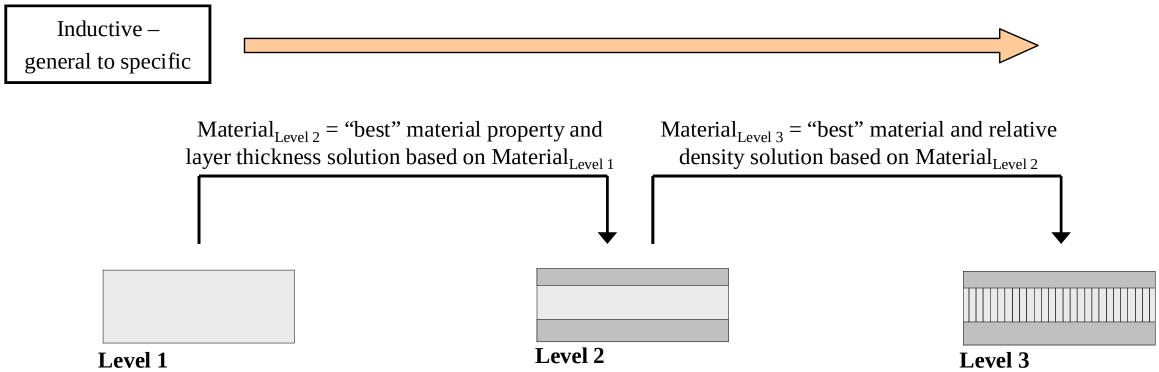


Figure 5.12 – Mapping functions of material properties in BRP design (inductive)

Inductive mapping functions

In order to achieve an inductive design solution, the solution path must travel from general to specific, that is, from Level 1 to Level 3 in BRP design. In an inductive solution path, design information from the previous level is preserved by imposing additional design constraints. For example, when making design decisions at Level 2, design information previously determined at Level 1 is preserved by constraining the design solution at Level 2 to have similar properties to the solution at Level 1 (a ± 5% deviation is acceptable in order to sufficiently relax the design constraint so a feasible design solution can be found). In BRP design deductive material mapping functions are imposed at Level 2 and Level 3, based on BRP design solutions determined at Level 1 and Level 2, respectively. By implementing these multilevel inductive design constraints, a designer is able to make design decisions at level of high complexity (such as Level 3) based on design information from Level 1 and Level 2. Inductive mapping functions for BRP design are described below.

- *Level 1 to Level 2* – Material properties and layer height corresponding to the three layers of a BRP designed at Level 2 are determined based on a robust design solution to achieve specified user defined goals. The limiting constraint is that the weighted average (based on material properties of each layer and height of layer compared to the total height of BRP) of the material properties of the three layers in Level 2 must equal the material properties in the single layer for the BRP at Level 1 (±5% acceptable deviation). See Equation 5.43 for mathematical representation.

- *Level 2 to Level 3* – The material properties and core geometry of the BRP at Level 3 is determined by finding a robust design solution to achieve user defined goals. The limiting constraint is that the relative density multiplied by the material properties of the core layer in Level 3 must equal the material properties of the core layer at Level 2 (±5% acceptable deviation). See Equation 5.44 for mathematical representation.

Uncertainty Mapping Functions

In order to quantify and model propagated uncertainty in a multilevel design process, inductive and deductive uncertainty mappings are developed. A description of uncertainty mappings in the multilevel BRP design problem is given in Figure 5.13. For the BRP design problem explored in this thesis, uncertainty is modeled as uncertainty in material properties [density (ρ), yield strength (σ_y)] and loading conditions [average peak pressure (p_0), characteristic loading time (t_0)]. Uncertainty is modeled as bounds surrounding an assumed nominal value.

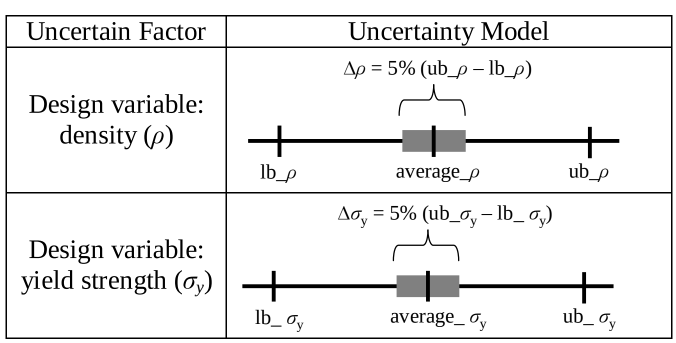


Figure 5.13– Uncertainty mapping in BRP design

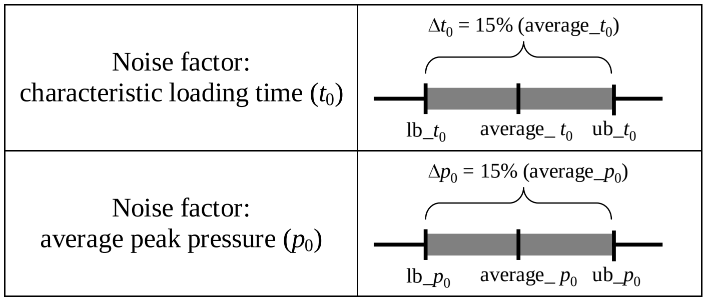


Figure 5.13 (continued) – Uncertainty mapping in BRP design

In the BRP design problem, it is assumed that material properties and loading conditions are uncertain. Uncertainty in loading conditions is considered a noise factor (Type I uncertainty) and uncertainty in material properties is categorized as uncertainty in a control factor (Type II uncertainty). In the BRP design problem, all uncertainty is modeled as an interval surrounding a nominal value. There is an equal probability for selecting any value within a specified uncertainty interval. Both inductive and deductive uncertainty mappings in the BRP design problem are defined as bounds on loading conditions and bounds on material properties relevant for design at a given level.

Find

Design variables and deviation variables at each level of design are determined for the robust solution, and are listed in Table 5.8.

Table 5.8 – Design variables and deviation variables to find in BRP design solution

Design Level	Design Variables	Deviation Variables
Level 1	σ_y, ρ, h	d_i^+, d_i^-
Level 2	$\sigma_{yf}, \sigma_{yc}, \sigma_{yb}, \rho_f, \rho_c, \rho_b, h_f, H, h_b$	d_i^+, d_i^-
Level 3	$\sigma_{yf}, \sigma_{yc}, \sigma_{yb}, \rho_f, \rho_c, \rho_b, h_f, H, h_b, h_c, B$	d_i^+, d_i^-

Satisfy

Constraints

For BRP design, performance constraints are given in Equation 5.36 – Equation 5.37.

$$\begin{aligned} &\delta_{\max} + \Delta\delta_{\max} \le 0.1L \\ &\delta_{\max} + \Delta\delta_{\max} \le 0.1 \quad \text{for } L = 1m \end{aligned} \tag{5.36}$$

$$M + \Delta M \le 150\, kg / m^2 \tag{5.37}$$

Material property constraints are given in Equation 5.38.

$$\begin{aligned} &(\text{lower bound}) - \left(x - \tfrac{1}{2}\Delta x\right) \le 0 \\ &\left(x + \tfrac{1}{2}\Delta x\right) - (\text{upper bound}) \le 0 \\ &\forall x = \left\{\rho_b, \rho_c, \rho_f, \sigma_{Y,b}, \sigma_{Y,c}, \sigma_{Y,f}\right\} \end{aligned} \tag{5.38}$$

Geometric constraints are given in Equation 5.39 – Equation 5.41.

$$\begin{aligned} &R_c = \left(2Bh_c - h_c^2\right) / B^2 \\ &0.07 - R_c \le 0 \end{aligned} \tag{5.39}$$

$$\begin{aligned} &g_{SH1} = \left(2 p_0 t_0 \big/ h_f \sqrt{\sigma_{Y,f}\,\rho_f}\right) - \Gamma_{SH}, \ \text{ where } \ \Gamma_{SH} = 0.6 \\ &\Delta g_{SH1} = \left|\frac{\partial g_{SH1}}{\partial p_0}\right| \Delta p_0 + \left|\frac{\partial g_{SH1}}{\partial t_0}\right| \Delta t_0 + \left|\frac{\partial g_{SH1}}{\partial \sigma_{Y,f}}\right| \Delta\sigma_{Y,f} + \left|\frac{\partial g_{SH1}}{\partial \rho_f}\right| \Delta\rho_f \\ &g_{SH1} + \Delta g_{SH1} \le 0 \end{aligned} \tag{5.40}$$

$$
\begin{aligned}
&g_{SH2} = \frac{\rho_c H R_c}{\rho_f h_f} - \frac{4}{\sqrt{3}} \\
&\Delta g_{SH2} = \left|\frac{\partial g_{SH2}}{\partial \rho_f}\right| \Delta \rho_f + \left|\frac{\partial g_{SH2}}{\partial \rho_c}\right| \Delta \rho_c \\
&g_{SH2} + \Delta g_{SH2} \le 0
\end{aligned}
\tag{5.41}
$$

Constraints relating to deviation variables are given in Equation 5.42.

$$
\begin{aligned}
&d_i^+ \cdot d_i^- = 0 \ , \ d_i^+, d_i^- \ge 0 \\
&\{i = 1,2,3,4\}
\end{aligned}
\tag{5.42}
$$

Inductive design constraints are given in Equation 5.1 – Equation 5.2. Recall from the cantilever beam example in Chapter 4 that in an inductive design solution path, additional constraints are added to preserve design information from levels of decreasing complexity. When transitioning from Level 1 to Level 2, an additional constraint is added at Level 2 such that the average material properties of a BRP at Level 2 are within 5% of the material properties of the BRP designed at Level 1. This constraint is reflected in Equation 5.43. Additionally, when transitioning from Level 2 to Level 3, an additional constraint is added at Level 3 such that effective material properties of the core layer at Level 3 are within 5% of the material properties of the core layer at Level 2. This constrained is detailed in Equation 5.44. Recall that effective material properties at Level 3 are determined by multiplying base material properties by the relative density (R_c) of the core.

$$
\begin{aligned}
&\left(x_{f,level2}\frac{h_f}{h}\right) + \left(x_{b,level2}\frac{h_b}{h}\right) + \left(x_{c,level2}\frac{H}{h}\right) = (\pm 5\%)\, x_{level1} \\
&x = \{\sigma_y, \rho\}
\end{aligned}
\tag{5.43}
$$

$$R_c x_{c,level3} = (\pm 5\%) x_{c,level2}$$
$$x = \{\sigma_y, \rho\} \tag{5.44}$$

Goals

There are four goals involved in BRP design including two performance goals and two robustness goals: minimize deflection of back face sheet, minimize mass / area, maximize robustness with based on deflection goal, maximize robustness based on mass /area goal. The metric used for determining design robustness is HD-EMI. Recall that the HD-EMI metric is a measure of the distance from design space bounds divided by system performance variation. Since there are two performance goals in the BRP design problem (minimize deflection, minimize mass / area) the HD-EMI metric is maximized considering variation in deflection and variation in mass / area. In Equation 5.45 – Equation 5.47 HD-EMI calculations as they relate to design variables and performance goals at each design level are given. Notice that by increasing distance from feasible design space and decreasing variation in system performance, one is able to maximize HD-EMI.

$$\mathrm{HD-EMI}_{\delta,level1} = \frac{\min\left(\left|x - x_{boundary}\right|\right)}{\max(\Delta\delta)}$$
$$\mathrm{HD-EMI}_{M,level1} = \frac{\min\left(\left|x - x_{boundary}\right|\right)}{\max(\Delta M)} \tag{5.45}$$
$$\forall x = \{\sigma_y, \rho, h\}$$

$$\mathrm{HD-EMI}_{\delta,level2} = \frac{\min\left(\left|x - x_{boundary}\right|\right)}{\max\left(\Delta\delta\right)}$$
$$\mathrm{HD-EMI}_{M,level2} = \frac{\min\left(\left|x - x_{boundary}\right|\right)}{\max\left(\Delta M\right)} \tag{5.46}$$
$$\forall x = \left\{\sigma_{y,f}, \sigma_{y,b}, \sigma_{y,c}, \rho_f, \rho_b, \rho_c, h_f, h_b, H\right\}$$

$$\mathrm{HD-EMI}_{\delta,level3} = \frac{\min\left(\left|x - x_{boundary}\right|\right)}{\max\left(\Delta\delta\right)}$$
$$\mathrm{HD-EMI}_{M,level3} = \frac{\min\left(\left|x - x_{boundary}\right|\right)}{\max\left(\Delta M\right)} \tag{5.47}$$
$$\forall x = \left\{\sigma_{y,f}, \sigma_{y,b}, \sigma_{y,c}, \rho_f, \rho_b, \rho_c, h_f, h_b, H, B, h_c\right\}$$

Minimize

A deviation function in minimized in order to determine a design solution that best meets design goals. In this thesis, due to its ease of use, the Archimedian formulation of the deviation function is chosen. It consists of a weighted sum of the deviation variables, and the weights are chosen to reflect designer preferences such that they are all greater than or equal to zero and sum to unity.

Weighted Sum of Deviation Variables

The deviation function for the BRP design is shown in Equation 5.48. The goal is to minimize the value of Z in order to find the design point with the smallest deviation from design goals. In BRP design, each goal is weighted equally at W_i = 0.25 (i = 1 to 4).

$$Z = W_1 \cdot d_1^- + W_2 \cdot d_2^- + W_3 \cdot d^3 + W_4 \cdot d^4 \tag{5.48}$$

5.2.3 Blast Resistant Panel Inductive Design Solution

The inductive multilevel BRP design solution is presented in Section 5.3.3. Following a general-to-specific (inductive) approach, the design solutions are presented based on a top-down approach: Level 1, Level 2, and Level 3.

Level 1

When obtaining an inductive multilevel design solution, the designer begins at Level 1, the level with the least amount of design complexity. The BRP design solution and performance data at design Level 1 are presented in Table 5.9 – Table 5.10.

Table 5.9 – BRP design variable data: Level 1

Design Variable	Value	Units
σ_y	1075	MPa
ρ	2941	kg/m^3
h	36	mm

Table 5.10 – BRP performance data: Level 1

Performance	Value	Units
back face sheet deflection (δ)	5.9	cm
mass / area (M)	107	kg/m^2
variation of deflection ($\Delta\delta$)	2.8	cm
variation of mass / area (ΔM)	43	kg/m^2

Level 2

Once the robust design solution is obtained for Level 1, an additional design constraint characterizing BRP material properties at Level 1 is imposed for BRP design at Level 2. The BRP design solution and performance data at design Level 2 are presented in Table 5.11 – Table 5.12.

Table 5.11 – BRP design variable data: Level 2

Design Variable	Value	Units
$\sigma_{y,f}$	983	MPa
$\sigma_{y,b}$	983	MPa
$\sigma_{y,c}$	1075	MPa
ρ_f	3496	kg/m^3
ρ_b	3496	kg/m^3
ρ_c	2201	kg/m^3
h_f	11	mm
h_b	11	mm
H	25	mm

Table 5.12 – BRP performance data: Level 2

Performance	Value	Units
back face sheet deflection (δ)	5.1	cm
mass / area (M)	131	kg/m^2
variation of deflection ($\Delta\delta$)	2.1	cm
variation of mass / area (ΔM)	19	kg/m^2

Level 3

Once the robust design solution is obtained for Level 2, an additional design constraint characterizing the core layer of the BRP design at Level 2 is imposed for BRP design at Level 3. The BRP design solution and performance data at design Level 3 are presented in Table 5.13 – Table 5.14. Design solutions at Level 3 are considered the final multilevel inductive design solution.

Table 5.13 – BRP design variable data: Level 3

Design Variable	Value	Units
$\sigma_{y,f}$	1064	MPa
$\sigma_{y,b}$	1063	MPa
$\sigma_{y,c}$	1075	MPa
ρ_f	3163	kg/m^3
ρ_b	3156	kg/m^3
ρ_c	2384	kg/m^3

Table 5.13 (continued) – BRP design variable data: Level 3

h_f	12	mm
h_b	12	mm
H	26	mm
B	7	mm
h_c	5	mm

Table 5.14 – BRP performance data: Level 3

Performance	**Value**	**Units**
back face sheet deflection (δ)	1.5	cm
mass / area (M)	131	kg/m^2
variation of deflection ($\Delta\delta$)	0.9	cm
variation of mass / area (ΔM)	19	kg/m^2

Robust vs. Non-Robust BRP Design Solutions

In the following section, the robustness of the multilevel BRP design solution is examined. Recall that the performance of a robust design is insensitive to slight variation in input parameters. The motivation for this section is to examine the differences in BRP robust vs. non-robust design (based on goal weighting factors). In the following tables, BRP design data from robust vs. non-robust design scenarios is compared for design solutions at each level. Additionally, the robustness of design solutions (based on the HD-EMI robustness metric) is analyzed at each level. Data supporting the observations in this section are given in Table 5.15 – Table 5.20. Recall that there are four design goals in BRP multilevel robust design: (1) minimize deflection (2) maximize robustness with respect to variation in deflection, (3) minimize mass / area, and (4) maximize robustness with respect to variation in mass / area. For a robust BRP design scenario, each goal is weighted equally at W_i = 0.25. For a non-robust design scenario, performance goals are weighted at $W_{\text{performance}}$ = 0.5 while the two remaining robustness goals are weighted at W_{robust} = 0.

In Table 5.15, robust and non-robust BRP design performance at Level 1 is compared.

Table 5.15 – Robust vs. non-robust BRP performance at Level 1

Performance	**Robust**	**Non-Robust**	**Units**
back face sheet deflection (δ)	5.9	6.5	cm
mass / area (M)	107	104	kg/m^2
variation of deflection ($\Delta\delta$)	2.8	3.2	cm
variation of mass / area (ΔM)	43	46	kg/m^2

In Table 5.15, variation in BRP performance is less for the robust scenario compared to the non-robust scenario, as expected. However, the performance goals of minimizing deflection and mass / area are also more closely achieved in the robust design scenario, a trend that is not expected.

In Table 5.16 the HD-EMI metric is calculated for the robust and non-robust design scenarios at Level 1. Recall that there are two components to the HD-EMI metric: the maximization of distance to design space bounds divided by the minimization of performance variation. Therefore, by maximizing HD-EMI, one is able to maximize system robustness. By comparing HD-EMI calculations for each design variable one is able to asses the relative robustness of BRP design at Level 1. In Table 5.16, the greatest HD-EMI value for each design variable is presented in bold type in an effort to compare robust and non-robust design scenarios.

Table 5.16 – Robust vs. non-robust BRP design solution at Level 1

Design Variable	**Design Solution**			**Robustness Metric**			
				HD-EMI$_\delta$ (m^{-1})		**HD-EMI$_M$ (m^2/kg)**	
	Robust	**Non-Robust**	**Units**	**Robust**	**Non-Robust**	**Robust**	**Non-Robust**
σ_y	1075	863	MPa	89	**741**	0.06	**0.52**
ρ	2941	2715	kg/m^3	**420**	279	**0.27**	0.19
h	36	38	mm	**1114**	1042	**0.73**	0.72

As seen in Table 5.16, the greatest HD-EMI values are most often found in the robust design scenario. This observation combined with the fact that performance variation is minimized in the robust design case (Table 5.15) indicates that for Level 1, the robust design case (W_i = 0.25) is indeed more robust than the non-robust case ($W_{\mathrm{performance}}$ = 0.5; W_{robust} = 0).

In Table 5.17, BRP design performance at Level 2 for the robust and non-robust cases is compared.

Table 5.17 – Robust vs. non-robust BRP performance at Level 2

Performance	**Robust**	**Non-Robust**	**Units**
back face sheet deflection (δ)	5.1	5.5	cm
mass / area (M)	131	130	kg/m^2
variation of deflection ($\Delta\delta$)	2.1	2.3	cm
variation of mass / area (ΔM)	19	20	kg/m^2

Similar to BRP performance at Level 1 (Table 5.15) variation in BRP performance at Level 2 (Table 5.17) is less for the robust case compared to the non-robust case, as expected. The BRP deflection goal at Level 2 is best achieved for the robust case, whereas the BRP mass / area goal is best achieved for the non-robust case. This shift in design trends may indicate that the differences in the robust and the non-robust design scenarios become less obvious at later stages in the BRP multilevel design process.

In Table 5.18, the robustness of BRP design scenarios at Level 2 is assessed using the HD-EMI metric. Similar to Table 5.16, the greatest HD-EMI value for each design variable is presented in bold type in an effort to compare robust and non-robust design scenarios.

Table 5.18 – Robust vs. non-robust BRP design solution at Level 2

Design Variable	Design Solution			Robustness Metric			
				HD-EMI$_\delta$ (m^{-1})		HD-EMI$_M$ (m^2/kg)	
	Robust	Non-Robust	Units	Robust	Non-Robust	Robust	Non-Robust
$\sigma_{y,f}$	983	750	MPa	557	**1522**	0.62	**1.75**
$\sigma_{y,b}$	983	750	MPa	557	**1522**	0.62	**1.75**
$\sigma_{y,c}$	1075	919	MPa	119	**787**	0.13	**0.91**
ρ_f	3496	3115	kg/m^3	**890**	606	**0.98**	0.70
ρ_b	3496	3115	kg/m^3	**890**	606	**0.98**	0.70
ρ_c	2201	2200	kg/m^3	**120**	109	0.13	0.13
h_f	11	12	mm	1984	**1993**	2.19	**2.29**
h_b	11	12	mm	1984	**1993**	2.19	**2.29**
H	25	26	mm	**2116**	2029	**2.34**	2.33

As seen in Table 5.18, instances of maximum HD-EMI occur in both the robust and non-robust case. This indicates that at Level 2 the measure of design robustness for the robust and non-robust design scenarios is similar. At Level 1 it is clear that the robust scenario produces a more robust design solution. However, at Level 2 the robust and non-robust design scenarios produce design solutions with similar levels of robustness.

In Table 5.19, BRP design performance at Level 3 for the robust and non-robust cases is compared.

Table 5.19 – Robust vs. non-robust BRP performance at Level 3

Performance	Robust	Non-Robust	Units
back face sheet deflection (δ)	1.5	1.6	cm
mass / area (M)	131	132	kg/m^2
variation of deflection ($\Delta\delta$)	0.9	1	cm
variation of mass / area (ΔM)	19	17	kg/m^2

Similar to Level 2 BRP performance, there is little difference in BRP performance at Level 3. Performance goals are slightly improved for the non-robust case compared to the robust case, as expected. Variation in performance is similar for the robust and non-robust cases.

In Table 5.20, the robustness of BRP design scenarios at Level 3 is compared by calculating the HD-EMI metric. Similar to Table 5.16 and 5.18, the greatest HD-EMI value for each design variable is presented in bold type in an effort to compare robust and non-robust design scenarios.

Table 5.20 – Robust vs. non-robust BRP design solution at Level 3

Design Variable	**Design Solution**			**Robustness Metric**			
				HD-EMI$_\delta$ (m^{-1})		**HD-EMI$_M$ (m^2/kg)**	
	Robust	**Non-Robust**	**Units**	**Robust**	**Non-Robust**	**Robust**	**Non-Robust**
$\sigma_{y,f}$	1064	874	MPa	400	**2260**	0.19	**1.33**
$\sigma_{y,b}$	1063	875	MPa	411	**2250**	0.19	**1.32**
$\sigma_{y,c}$	1075	1024	MPa	278	**760**	0.13	**0.45**
ρ_f	3163	3624	kg/m^3	1615	**2030**	0.77	**1.19**
ρ_b	3156	3624	kg/m^3	1606	**2030**	0.76	**1.19**
ρ_c	2384	2326	kg/m^3	**533**	408	**0.25**	0.24
h_f	12	12	mm	**5093**	4583	2.41	**2.70**
h_b	12	12	mm	**5093**	4583	2.41	**2.70**
H	26	26	mm	**5185**	4667	2.46	**2.75**
B	7	10	mm	3509	**4737**	1.66	**2.79**
h_c	5	5	mm	**5499**	4949	2.60	**2.91**

As seen in Table 5.20 the non-robust design solution at Level 3 contains many design variable solutions that are more robust than correlating design variable solutions for the robust case. This unexpected outcome is especially true when considering HD-EMI calculations relating to the mass / area performance goal. Why does employing a non-robust weighting scheme produce more robust solutions than employing a robust weighting scheme at several iterations into a multilevel design process? Besides the change in goal weighting, the only difference in the robust and non-robust design process is in the mapping functions. At the beginning of BRP design, a design solution is found at Level 1. Recall that at this design level a robust weighting scheme produces the most robust design solution. Then, design information from the robust and non-robust Level 1 solution is input into Level 2 mapping function constraints. Since the robust and non-robust weighting schemes at Level 1 product different design solutions, the mapping functions are slightly different for robust and non-robust design at Level 2. Next, a design solution is obtained for Level 2 design using the two design scenarios. At this stage in the design process, it is observed that there is little difference in the robustness of design solutions obtained from the robust and non-robust weighting schemes. Again, design information from Level 2 is input as mapping function constraints into Level 3 and design solutions are obtained at Level 3 based on the specified goal weighting schemes. At Level 3 design, the observed trend continues and the design solution from the non-robust weighting scheme actually produces more robust results than the design solution from the robust weighting scheme. While this unexpected trend is not fully understood at this time, it is almost certainly related to the mapping functions used to transfer design information among each level since the solution-finding approach for the robust and non-robust case is identical, except for goal weighting and mapping functions.

In summary, early in the BRP design process (Level 1) design solutions obtained from a robust goal weighting scheme are more robust than design solutions achieved based on non-robust goal weighting. As the designer progresses through the design process (Level 2), the robust and non-robust design solutions begin to have similar levels or robustness. Then, late in the design process (Level 3) design solutions from a non-robust weighting scheme are actually more robust than design solutions from a robust weighing scheme. This unexpected trend in multilevel design is attributed to the mapping functions which control what information is passed to subsequent design decisions and set the tone for design solutions later in the design process. Based on design information presented in this section, should multilevel robustness be omitted from multilevel design processes because it is unpredictable and difficult to attain? No, if anything, the previous section serves to illustrate the significant impact of propagated process chain uncertainty introduced by mapping functions in multilevel design, indicating that there remains a great need for robustness in multilevel design. Information presented in the previous section reiterates the impact of mapping functions in reaching desirable multilevel design solutions. Based on the previous section, with the incorporation of a multilevel robustness approach in multilevel design, the designer is encouraged to devote significant effort in mapping function design as a key component of a multilevel design process.

Inductive vs. Deductive BRP Design Approach

In the following section, BRP design solutions attained from an inductive multilevel design approach and a deductive design approach are compared. Recall that in an inductive design approach design decisions are made in a top-down or general-to-specific manner. In BRP multilevel inductive design, design decisions are first made at the simplest design level (Level 1). Then design information from general models is used in making more specific or complex design decisions. In a deductive design approach, design decisions are made in a bottom-up or specific-to-general way. In BRP deductive design, a BRP design solution is reached at Level 3 without the addition of design information from more general models. With 11 design variables, BRP design is sifficiently complex to benefit from a multilevel design approach without necessitating its use. BRP design at Level 3 can be directly solved using analytical equations describing BRP performance and a computational solution-finding algorithm. In Table 5.21 and Table 5.22 BRP Level 3 design information is compared for a deductive and inductive design solution approach.

Table 5.21 – Inductive vs. deductive BRP performance at Level 3

Performance	Deductive	Inductive	Units
back face sheet deflection (δ)	1.9	1.5	cm
mass / area (M)	133	131	kg/m^2
variation of deflection ($\Delta\delta$)	1.2	0.9	cm
variation of mass / area (ΔM)	17	19	kg/m^2
Relative density of core (R_c)	0.67	0.73	unitless

Table 5.22 – Inductive vs. deductive BRP design solution at Level 3

Design Variable	Deductive	Inductive	Units
$\sigma_{y,f}$	614	1064	MPa
$\sigma_{y,b}$	689	1063	MPa
$\sigma_{y,c}$	612	1075	MPa
ρ_f	3758	3163	kg/m^3
ρ_b	3582	3156	kg/m^3
ρ_c	2200	2384	kg/m^3
h_f	12	12	mm
h_b	13	12	mm
H	27	26	mm
B	12	7	mm
h_c	5	5	mm

As seen in Table 5.21, in general, BRP design goals are more closely achieved when using an inductive design approach. However, BRP performance for an inductive design approach and direct calculation are similar. Comparing BRP design solutions from Table 5.22, it is observed that material property data (σ_y) and geometric data (B) for the two design scenarios are significantly different. Since each design scenario produces similar BRP performance it is concluded that differences in design variable data indicate different by comparable solutions in BRP design space.

In summary, the above design comparison indicates that while implementing an inductive and deductive solution path in BRP design produces different design solutions, the design solutions measure remarkably similar performance. This realization adds value to the inductive design approach. In complex BRP design, an inductive design approach simplifies the design process by limiting the design space at complex design levels. Based on the information in the previous section, it is observed that an inductive solution path in which complex design space is limited does not inversely affect the overall performance of the attained BRP design solution.

5.3 VERIFICATION AND VALIDATION BASED ON BLAST RESISTANT PANEL DESIGN

The following section contains evidence for the verification and validation of the template-based approach to multilevel design presented in Chapter 3 by considering the design of blast resistant panels. First, the domain-specific structural validity is examined by investigating the appropriateness of the BRP design problem in adding value to the verification and validation of the multilevel design template. Then, the domain-specific performance validity of the multilevel design template is examined in by looking at the solutions obtained from completing the BRP example problem in Chapter 5. Appendix D contains information regarding the validation of computational design tools used in solving the BRP design problem.

Recall the Validation Square discussed in detail in Chapter 2. To summarize, the Validation Square is a construct used in the verification and validation of design methods. In an attempt to facilitate a systematic approach for the validation of design methods, the Validation Square is divided into four sections. Recall that the sections of the Validation Square dealing with the application of the proposed design method to example problems include *domain-specific structural validity* and *domain-specific performance validity*. In the following sections, ways in which completing the BRP example problem add value to the domain-specific structural validity and domain-specific performance validity of the developed multilevel design template are presented.

Both the cantilever beam example problem (Chapter 4) and the BRP example problem (Chapter 5) are used in testing the domain-specific structural validity and domain-specific performance validity of the developed multilevel design template. The cantilever beam example problem is chosen because of the simplified nature of the problem. Initially, the cantilever beam example problem is used to refine the developed multilevel design template. After the final template is developed, the cantilever beam example problem is used to illustrate the concepts of template base multilevel robust design in a rather idealized environment. However, the domain-specific structural validity and domain-specific performance validity are not satisfied by considering only the simplified cantilever beam example problem. The BRP example problem is included in this thesis because it is used to illustrate the effectiveness of a template-based approach to multilevel robust design in a more complex design environment. The results of the cantilever beam example problem and the BRP example problem combine to provide domain-specific structural validity and domain-specific performance validity to the developed multilevel design template.

5.3.1 Domain-Specific Structural Validity

Domain-specific structural validity relates to the appropriateness of the selected example problem and the designer is prompted to ask the question "Is the example problem used in demonstrating the method an appropriate choice?". It is asserted that the BRP example problem is an appropriate choice for testing the effectiveness of the developed multilevel design template for because the BRP problem possesses the following characteristics:

Clearly Defined Design Problem

The BRP design problem contains clearly defined design variables, bounds on design variables, constraints, goals, and preferences. Each of these descriptions is needed for the successful implementation of the multilevel design template. Additionally, material property mappings and uncertainty mappings are known or easily determined for the BRP design problem. The multilevel robust design temple is developed for a design environment in which design requirements, bounds, constraints, goals, and preferences are clearly known.

BRP Design is Multilevel in Nature

The BRP design problem is multilevel in nature. The separation of levels in the BRP design problem involved determining various levels of model complexity. The specified levels of model complexity are implemented at various levels in the multilevel design template. While single-level design problems could be solved using the multilevel design template (the designer would consider only one level in the design process) in order to test the full range of effectiveness of the multilevel design template, analysis of multilevel design problem is needed.

Phenomena in BRP Design at Various Levels Affect BRP Performance

Any design problem can be divided into various levels and labeled a multilevel design problem. However, it is important that phenomena affecting product performance occurs at each level and is captured in descriptive models at each level. Otherwise, the cost of analyzing a design problem at multiple levels does not yield any design benefit. In other words, dividing a design problem into various levels should be done for a reason. If the designer is not interested in or cannot model the relevant phenomena at a specific level, then that particular level should not be included in design analysis because no value is added. For the BRP design problem, each level of model complexity considered contained valuable information relating the complexity of a performance model to the accuracy of a performance prediction. Therefore, not only is the BRP a multilevel design problem, be each level considered adds value to the overall BRP product design process.

Uncertainty in BRP Design Problem is Characterized

All design problems contain uncertainty. However, in order to design a robust solution, the uncertainty in a design problem must be sufficiently characterized. In the BRP design problem, there are numerous sources of uncertainty. In an attempt to account for design uncertainty without over-complicating the design process, uncertainty in the BRP design problem is modeled as uncertainty in loading conditions and uncertainty in material properties. Because models describing the uncertainty in the BRP design problem are developed, robust design techniques can be employed to develop a robust design solution. Robust design is a key element in the developed multilevel design template. The BRP example problem is appropriate choice in testing this aspect of the developed design template.

5.3.2 Domain-Specific Performance Validity

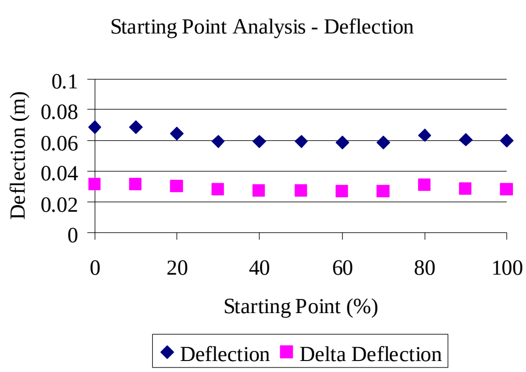
Domain-specific structural validity relates to the outcome of applying the method to an example problem and is used to ask the question "Does the application of the method to the example problem produce useful results?". To adequately address this question two topics are considered: the usefulness of the numerical results and the overall usefulness of the multilevel design template.

Appropriateness of BRP Design Solutions

When solving a BRP robust design problem, the results obtained are reasonable. That is, the solution to design variables and the predicted BRP performance agree with what is intuitively expected. However, due to the complex nature of BRPs, many aspects of design performance are too complicated to follow a designer's intuition. Based on a comparison of BRP performance results given in Section 5.5, it is shown that BRP performance predictions at various levels of model complexity are within an acceptable range of error. Also, BRP performance data at each level of model complexity agrees with BRP performance equations detailed in the work of Hutchinson and Xue, 2005. For future work, it is beneficial to compare BRP performance measurements calculated using analytical models with BRP performance determined using a finite element analysis computational tool.

Starting Point Analysis for BRP Multilevel Design

The internal consistency of the numerical BRP design solution is also tested with a starting point analysis. The BRP example problem is solved using an optimization routine at each design level. Therefore, it is important to determine if the selected starting point at each level results in a robust, stable solution that most closely meets design goals. A starting point analysis that implemented ten different starting points is completed for each design level. The starting points are at 10% increments of the design variable bounds. BRP performance is measured at each starting point. The starting point analysis for each level of BRP design is given in Figure 5.14 – Figure 5.16.



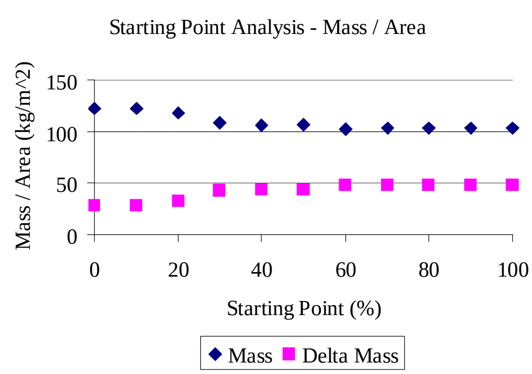


Figure 5.14 – BRP starting point analysis – Level 1

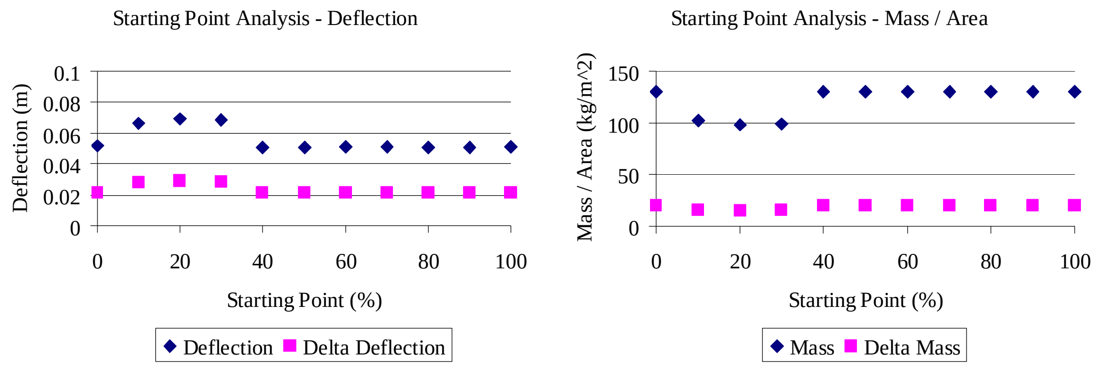


Figure 5.15 – BRP starting point analysis – Level 2

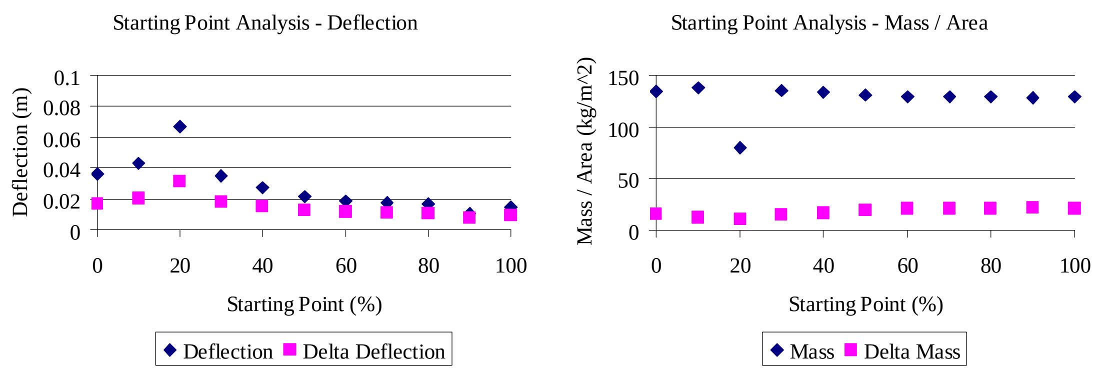
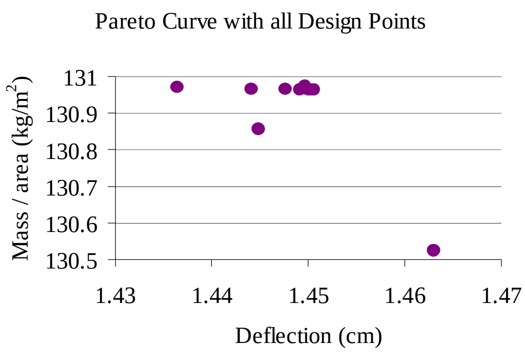


Figure 5.16 – BRP starting point analysis – Level 3

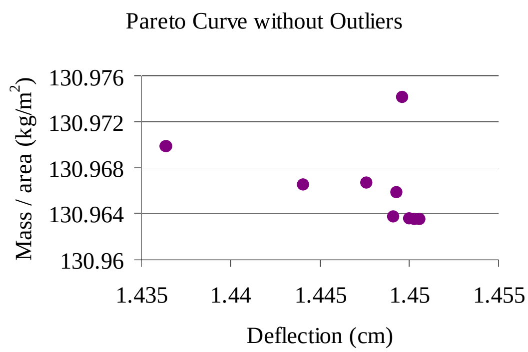
For BRP design at Level 1, all starting points result in a stable design solution. In BRP design at Level 2, the starting points that result in the most stable design solutions occur after 40% of design variable bounds. For BRP design at Level 3, starting at 20% of variable bounds results in an unstable design solution, and this starting point should be avoided. The starting point selected for design at each level is the midpoint of each design variable bound (50%). As seen in the previous figures, this results in stable BRP performance at each design level.

Pareto Curves for BRP Multilevel Design

Recall that for BRP design there are four goals: (1) minimize deflection, (2) maximize robustness considering variation in deflection, (3) minimize mass / area, and (4) maximize robustness considering variation in mass / area. For BRP design solutions, each goal is weighted equally at W_i = 0.25. Additional goal weighting schemes are employed to determine Pareto curves for BRP design at Level 3 in order to identify the relationships between the four goals. Eleven weighting schemes are used to compare two design goals such that the goal weighting for the first goal varies from 0 to 1 at increments of 0.1, while goal weighting for the second goal varies from 1 to 0 at increments of 0.1 (while all remaining goals held constant at zero). Pareto curves in Figure 5.17 represent all possible comparisons of BRP goals, examined in pairs [(a) M vs. δ, (b) $\Delta\delta$ vs. δ, (c) ΔM vs. M, (d) ΔM vs. δ, (e) $\Delta\delta$ vs. M, (f) ΔM vs. $\Delta\delta$]. Two graphs are given for each comparison including a graph displaying all eleven data points and a graph in which outliers are removed. Pareto graphs with outliers removed are used to observe the general relationship between the compared goals when extreme data points may hinder the realization of overall data trends. In Appendix E, data points with corresponding goal weighting scenarios are presented for each graph in Figure 5.17.

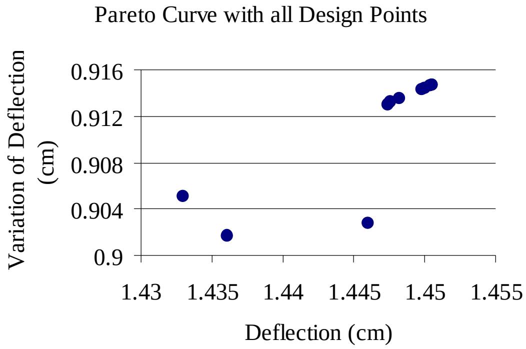


(a.1)

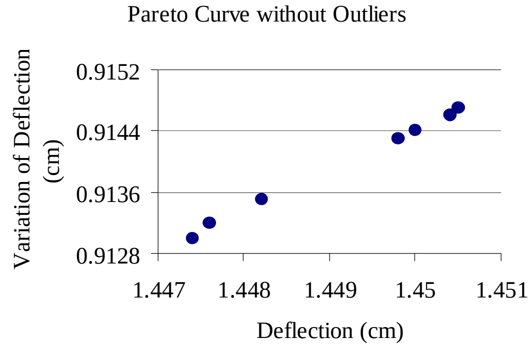


(a.2)

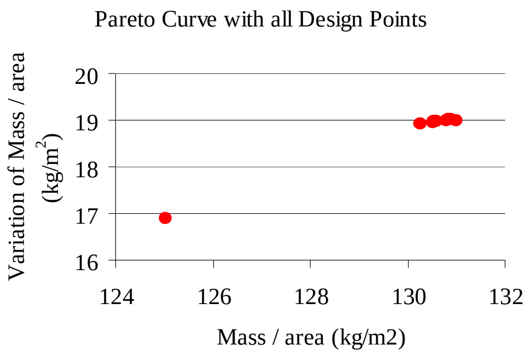
Figure 5.17 – BRP Pareto curves for Level 3 BRP design



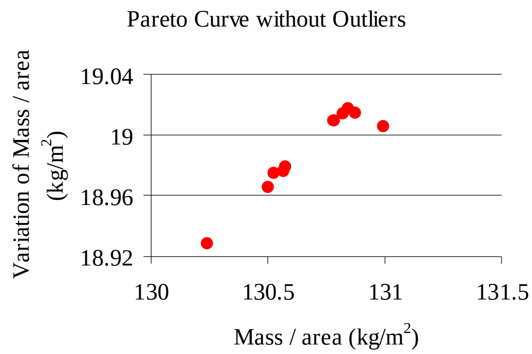
(b.1)



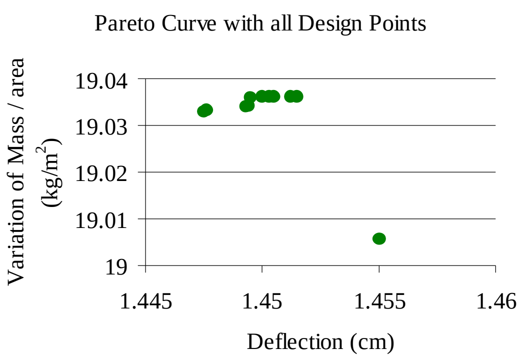
(b.2)



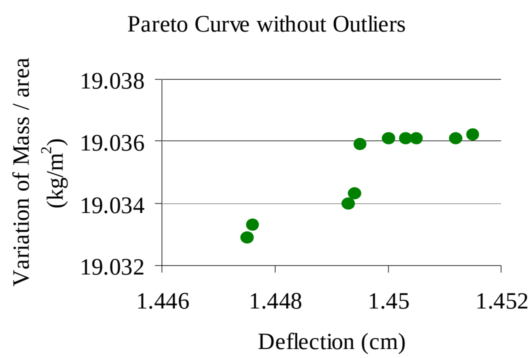
(c.1)



(c.2)

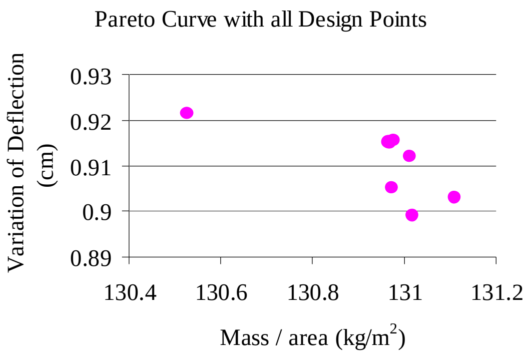


(d.1)

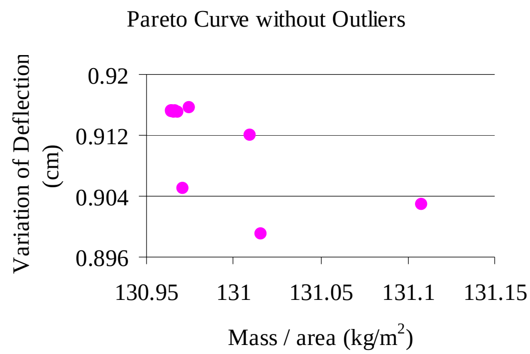


(d.2)

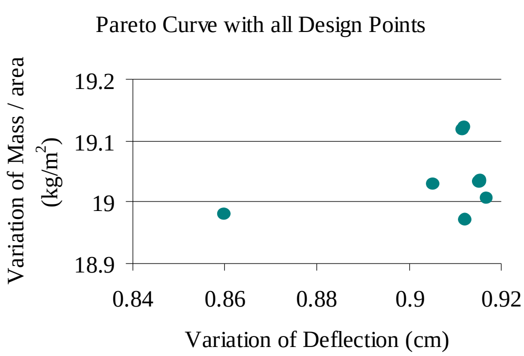
Figure 5.17 (continued) – BRP Pareto curves for Level 3 BRP design



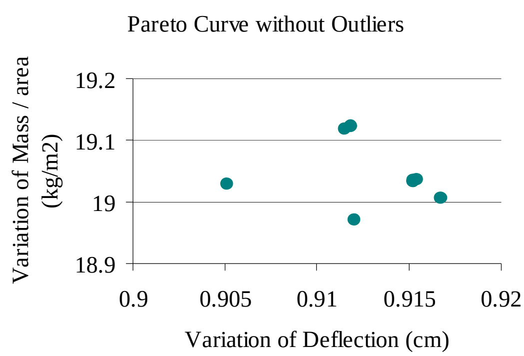
(e.1)



(e.2)



(f.1)



(f.2)

Figure 5.17 (continued) – BRP Pareto curves for Level 3 BRP design

In Figure 5.17 (a.1), (a.2) comparing mass / area vs. deflection, a slight inverse relationship is detected, best shown in graph (a.2) with outliers removed. That is, as mass / area decreases, panel deflection increases. This trend agrees with designer intuition and is observed in BRP design space exploration in that smaller deflection values can be obtained by increasing the mass / area constraint. Graphs (b.1), (b.2) and (c.1), (c.2) in Figure 5.17 display a comparison of variation in deflection vs. deflection and variation in mass / area vs. mass / area, respectively. For each of these comparisons, a loose direct relationship between BRP performance and BRP variation of performance is observed, illustrating that as performance increases, variation in performance also increases. As shown in Equation 5.30 and Equation 5.31, variation of performance is a function of BRP performance (δ, M). Therefore, it follows that a change in BRP performance would result in a change in variation in performance, and based in graphs (b.1), (b.2) and (c.1), (c.2) in Figure 5.17, it is shown that the compared quantities are directly related. In graphs (d.1), (d.2), (e.1), (e.2), (f.1), and (f.2) in Figure 5.17 no conclusive relationships are detected among the compared quantities. This indicates to the designer that there is little correlation between variation in mass / area vs. deflection, variation in deflection vs. mass / area, and variation in mass / area vs. variation in deflection.

Outliers in Figure 5.17 are not simply disregarded in Pareto curve analysis. In fact, these outliers may give the designer an indication of the "best" weighting scenarios for achieving design goals. In Appendix E, outliers are highlighted in grey and most often occur at weighing scenarios in which one goal is weighted significantly greater than another goal. In Table 5.23, outliers from the six Pareto graphs in Figure 5.17 are shown with goal weighting and BRP performance information included. Notice that, with the exception of outliers in rows 4 and 8, outliers tend to occur when one goal is weighted significantly greater than the other goal.

Table 5.23 – Pareto frontier data outlier analysis

Row	**Weight, Goal X**	**Weight, Goal Y**	**Performance X**	**Performance Y**
1	$W_\delta = 0.2$	$W_{\text{HD-EMI}\delta} = 0.8$	$\delta = 1.45$ cm	$\Delta\delta = 0.90$ cm
2	$W_\delta = 0.1$	$W_{\text{HD-EMI}\delta} = 0.9$	$\delta = 1.44$ cm	$\Delta\delta = 0.90$ cm
3	$W_\delta = 0$	$W_{\text{HD-EMI}\delta} = 1$	$\delta = 1.43$ cm	$\Delta\delta = 0.91$ cm
4	$W_M = 0.4$	$W_{\text{HD-EMI}M} = 0.6$	$M = 125.02$ kg/m^2	$\Delta M = 16.90$ kg/m^2
5	$W_\delta = 0$	$W_{\text{HD-EMI}M} = 1$	$\delta = 1.46$ cm	$\Delta M = 19.01$ kg/m^2
6	$W_M = 1$	$W_{\text{HD-EMI}\delta} = 0$	$M = 130.53$ kg/m^2	$\Delta\delta = 0.92$ cm
7	$W_\delta = 0$	$W_M = 1$	$\delta = 1.46$ cm	$M = 130.53$ kg/m^2
8	$W_{\text{HD-EMI}\delta} = 0.6$	$W_{\text{HD-EMI}M} = 0.4$	$\Delta\delta = 0.86$ cm	$\Delta M = 18.98$ kg/m^2

From the first three rows of data in Table 5.23, an interesting and unexpected design trend is observed. BRP design solutions with low values of deflection *and* low values of variation in deflection are observed when the robustness goal related to variation in deflection (HD-EMI$_\delta$) is weighted significantly higher than the deflection goal. Additionally, in row 4 it is observed that the mass / area and robust goal relating to mass / area are best achieved when the robustness goal is weighted slightly higher than the mass / area performance goal. Based on weighing schemes that best achieve design goals (row 3, row 4 in Table 5.23), a goal weighting scenario incorporating all four design goals is developed. The weighing scenario and design solution (compared to the design solution with design goals equally weighted at W_i = 0.25) is presented in Table 5.24.

Table 5.24 – Weighting scheme to best achieve design goals based on Pareto curves

SCENARIO A: Priority based on Pareto curves		SCENARIO B: Equal priority for each design goal	
Weight	Performance	Weight	Performance
$W_\delta = 0$	$\delta = 1.45$ cm	$W_\delta = 0.25$	$\delta = 1.45$ cm
$W_{\text{HD-EMI}\delta} = 0.5$	$\Delta\delta = 0.92$ cm	$W_{\text{HD-EMI}\delta} = 0.25$	$\Delta\delta = 0.91$ cm
$W_M = 0.2$	$M = 130.97$ kg/m^2	$W_M = 0.25$	$M = 130.97$ kg/m^2
$W_{\text{HD-EMI}M} = 0.3$	$\Delta M = 19.03$ kg/m^2	$W_{\text{HD-EMI}M} = 0.25$	$\Delta M = 19.03$ kg/m^2

The two weighting schemes in Table 5.24 yield similar BRP performance levels. However, notice that in Scenario A in Table 5.24 the deflection goal is not considered in reaching a design solution ($W_\delta = 0$) leading to the reasonable conclusion that the deflection goal has little effect in achieving BRP performance and robustness goals. Also, it is observed that the robustness goal related to variation in deflection is significant in achieving design solutions that best meet design goals. Such conclusions, which are beyond designer intuition, could be useful starting points when developing weighting schemes for BRP design at increased levels of model complexity.

Usefulness of Multilevel Template in BRP Design

Now that evidence supports the validity of the multilevel BRP design solutions, it is important to identify if the multilevel design template is useful in BRP design. Simply obtaining valid solutions from implementing the multilevel design template is not sufficient for it domain-specific performance validity. The benefits experienced from implementing the multilevel design template in BRP design should also be investigated.

The usefulness of applying the multilevel design template to the BRP example problem is demonstrated in decreased design complexity and computation cost observed in completing the example problem. As BRP model complexity decreased from a BRP modeled with three layers with a honeycomb core to a BRP modeled as a single panel, the number of design variables decreased from 11 to 3. Also, BRP performance calculations are significantly less complex for the BRP modeled at the lowest level of complexity compared to similar equations for a BRP modeled at the highest level of complexity considered in this design problem. This reduction in performance modeling is made possible by relatively simple mapping functions describing the relationship of material properties and uncertainty models throughout all levels in the BRP design problem. By applying mapping functions, the designer can travel (inductively and deductively) throughout all levels of the BRP design problem at low computation cost. To summarize, the template-based approach to multilevel robust BRP design is a valuable design strategy because the complexity of the design problem was decreased leading to agile design space exploration; and at low computation cost, mapping functions are used to travel throughout the multilevel design problem in both a top-down and bottom-up path. A visual representation of the value added to the verification and validation of the developed design template provided in Chapter 5 is shown in Figure 5.18.

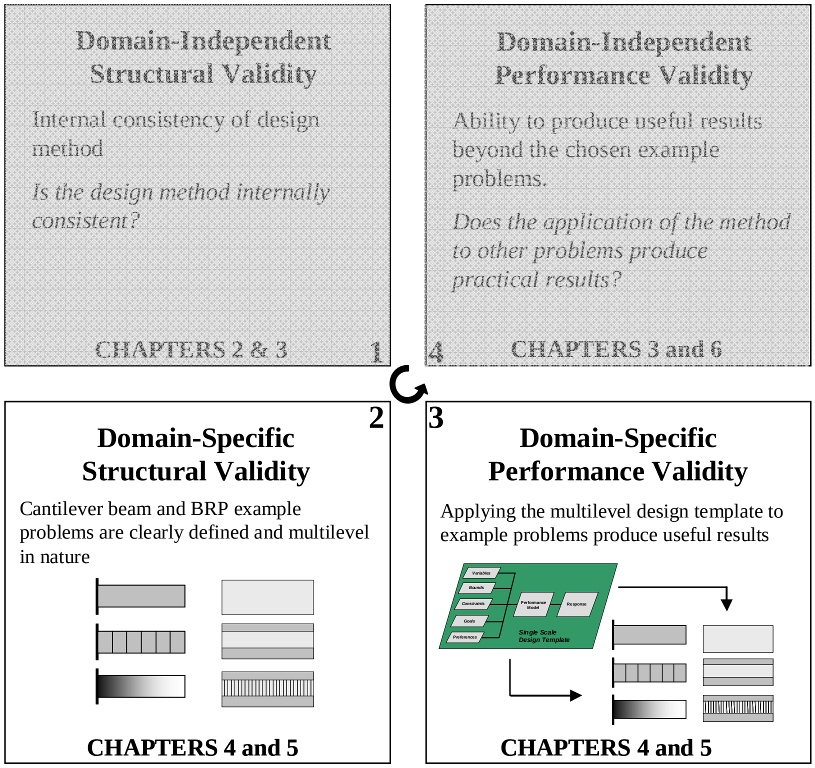


Figure 5.18 – Value added to verification and validation of design template – Chapter 5

5.4 SYNOPSIS OF CHAPTER 5

The completion of the BRP design problem adds value to the validation of the multilevel design template. It is shown that the developed design template can be applied to complex and uncertain design problems with desirable results. Recall that in Chapters 1 and 2 the context for this thesis is set by a discussion of multilevel design, design uncertainty, robust design, and template-based design. In Chapter 3, the multilevel design template, based on IDEM, is presented and discussed. In Chapters 4 and 5 the multilevel design template is applied to example problems in order to prove its logical flow of information and usefulness in the design environment. In Chapter 6, the thesis concludes with a summary of the validation of the multilevel design template based on the Validation Square. Closing comments relating to the general benefits of a template-based approach to multilevel design, as well as the intellectual contributions presented in this thesis are discussed.

CHAPTER 6

METHOD VALIDATION AND CLOSING STATEMENTS

Chapter 6 begins with a discussion of the intellectual contributions presented in this thesis, which include the development of a multilevel design template for achieving inductive, multilevel, robust design solutions and the application of the design template for multilevel BRP design. Next, the verification and validation of the multilevel design template is examined using the Validation Square construct. Components of method validation presented throughout this thesis are brought together in order to asses the overall validity of the multilevel design template. Chapter 6 concludes with a discussion of opportunities for improvement related to the information presented in this thesis. Future work related to the multilevel design template and the application of the multilevel design template in the example problems are discussed. Improvements in the multilevel design template and the example problems lead to a discussion of how a template-based design approach can gain increasing influence in the design community. At the conclusion of Chapter 6, lessons learned from developing and implementing the multilevel design template are discussed. A summary of the information presented in Chapter 6 is given in Table 6.1, and Chapter 6 in relation to the remainder of this thesis is illustrated in Figure 6.1.

Table 6.1 – Summary of Chapter 6

Heading / Sub-Heading	**Information**
Intellectual Contributions Based on Answering Research Questions	
Development of a Multilevel Design Template	Research contributions include: - Adapting IDEM to a template-based environment - Multilevel defined by levels of model precision / complexity
Application of Design Template to Multilevel Design of BRPs	Multilevel design template is particularized for BRP design and a multilevel robust BRP design solution is obtained

Table 6.1 (continued) – Summary of Chapter 6

Verification and Validation of Multilevel Design Template	
Quadrant 1 - DISV	Is the multilevel design template internally consistent?
Quadrant 2 - DSSV	Are the example problems appropriate choices?
Quadrant 3 - DSPV	Does the application of the multilevel design template to the example problems produce useful results?
Quadrant 4 - DIPV	Can it be assumed that the multilevel design template could be applied to additional example problems with positive results?
The Validation Square – Bringing It All Together	Confidence built in the overall validation of the multilevel design template based on 4 quadrants of the Validation Square
Opportunities for Improvement	
Future Work Related to Multilevel Design Template	- Particularizing the generic multilevel design template for example problems - A closer look model complexity in multilevel design - New methods for incorporating robustness in multilevel design template
Future Work Related to Example Problems	- Future work for cantilever beam example problem - Future work for BRP example problem
Vision for Template-Based Engineering Design of the Future	Opportunities for advancement in template-based multilevel robust design of the future: - Collaborative, distributive template-based design - Design template database
Lessons leaned	
Chapter 6 Synopsis	

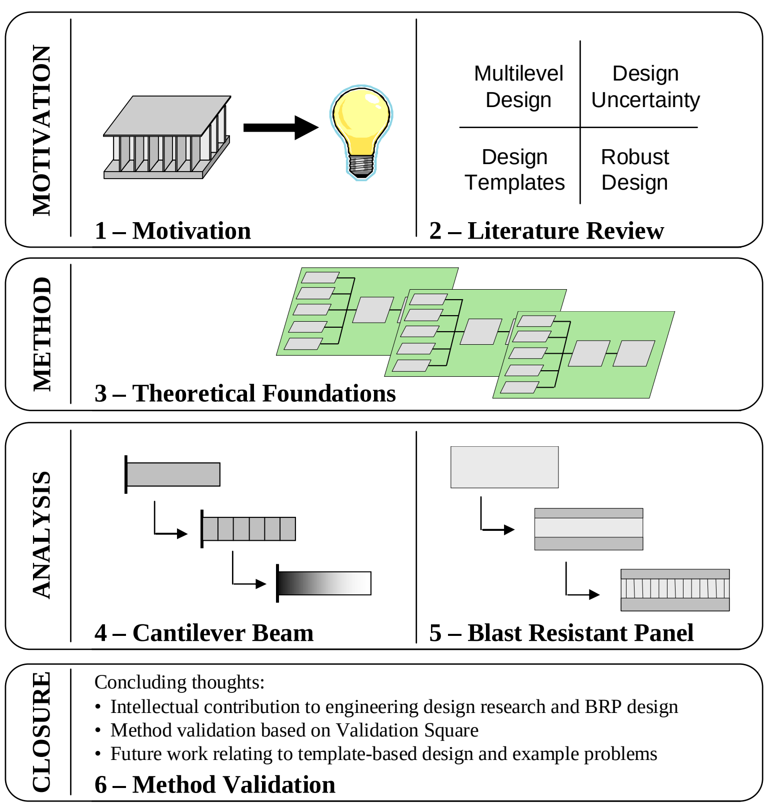


Figure 6.1 – Setting the context for Chapter 6

6.1 INTELLECTUAL CONTRIBUTIONS BASED ON ANSWERING RESEARCH QUESTIONS

In Section 6.1, the key research contributions of this thesis are discussed. The research contributions are divided into two categories corresponding to the primary and secondary research hypotheses. Key research contributions in this thesis include the development of a multilevel design template (Section 6.1.1), and the application of the robust design template in multilevel robust BRP design (Section 6.1.2). An overview of research contributions in this thesis is presented in Table 6.2.

Table 6.2 – Research contributions of this thesis

Research contributions in template-based multilevel design (Primary RQ)	
Added value to the study of multilevel robust design by combining it with concepts of template-based design	§3.1, §3.2
Defined design problems according to levels of model complexity or resolution in multilevel design and analysis	§1.1, §2.1
Developed multilevel design template based on existing design tools: the cDSP and IDEM	§2.3.3, §3.1, §3.2
Infused a multiscale robust design approach in the multilevel design template	§2.2.3, §3.1, §3.2
Compiled a glossary of key terms relating to template-based multilevel robust design	Glossary of Key Terms at beginning of thesis
Verified multilevel design template using the Validation Square	§3.3, §6.2
Research contributions in BRP design (Secondary RQ)	
Illustrated the multilevel nature of the BRP problem, and divided it according to levels of model complexity	§5.1
Particularized the multilevel design template for BRP design	§3.2.3, §5.2.1
Developed BRP performance models for each level, implemented performance models in MATLAB	§5.2.2, Appendix C
Developed mapping functions linking BRP design information among various levels in multilevel BRP design	§5.2.2
Adapted a multilevel robust design method (IDEM) for the robust multilevel design of BRPs	§3.1.3, §5.2.2, §5.2.3
Determined inductive BRP design solution robust to variation in material properties and loading conditions	§5.2.3

6.1.1 Development of a Multilevel Design Template

Recall the primary research question and hypothesis addressed in this thesis from Section 1.3.1 which is restated below:

Primary Research Question

How can information regarding multilevel robust design processes be captured and stored in a reusable format?

Primary Research Hypothesis

Information regarding robust multilevel design processes can be captured and stored in a reusable format by developing generic, reusable, computer executable design templates based on the Compromise Decision Support Problem (cDSP) and Inductive Design Exploration Method (IDEM).

The primary research hypothesis outlines the key research contribution in this thesis—the development of a template-based approach to multilevel robust design. In response to the primary research question, a multilevel design template is created in order to capture and store information from a multilevel design process in a reusable format. The unique research contributions inherent in the multilevel design template are the transformation of an existing design method to a template-based design environment and the categorization of a multilevel design process based on degrees of model complexity. These unique research contributions are discussed in the following paragraphs.

Adapting an Existing Design Method for a Template-Based Design Approach

The multilevel design template is adapted from two existing design constructs: the cDSP and IDEM. Recall that IDEM is a multilevel robust design method which, in its original form, is not well-adapted for a template-based design environment (Choi, et al. 2005). A key research contribution in this thesis is the adaptation of IDEM to a multilevel design template capable of collecting and storing design information in a multilevel design process. The multilevel design template is based on the structure of the cDSP, which is organized under the headings of *Given*, *Find*, *Satisfy*, and *Minimize*. In order to capture the procedural steps of IDEM in a design template construct based on the cDSP, two additional headings (*Define* and *Map*) are added. The multilevel design template provides a generic framework for multilevel design problems. The multilevel design template can be particularized for a variety of multilevel design problems in order to guide a designer in multilevel decision-making. Additionally, a multilevel robustness goal (based on IDEM) is easily incorporated in the multilevel design template in order to achieve multilevel design solutions that are robust to propagated process chain uncertainty.

Defining Complex Systems According to Levels of Model Complexity

Another key research contribution realized in the development of a multilevel design template is a method of dividing a complex design problem according to levels of model precision. In the materials design community, multiscale material modeling and design processes are divided according to material behavior at different length scales, or time scales. In this thesis, multiscale modeling according to length or time scales is extended beyond the material modeling community to include the division of complex design problems based on model precision or complexity. Prediction models with high precision are often characterized by high complexity, and low precision models tend to be less complex. In this thesis, complex multilevel design problems are divided according to levels of model abstraction in order to facilitate agile design space exploration by using approximation models of complex system behavior. This contribution involves adapting a proven modeling technique from the materials design community (multiscale modeling) and abstracting it for application in a broad range of complex systems design.

Limiting Design Space Using Mapping Functions

An additional intellectual contribution presented in this thesis is a framework to achieve satisficing design solutions to complex design problems by limiting design space at complex design levels via mapping functions. For complex multilevel design problems with many design variables, it is impractical to thoroughly explore the entire design space in order to find the optimum design solution. One of the key advantages of the multilevel design template is that it is used to identify regions of the design space for which a satisficing design solution is likely to exist. The multilevel template-based design procedure begins by determining a design solution at the simplest level of model complexity, Level 1. Then, design information from Level 1 is passed to the level with slightly increased model complexity, Level 2, via a predetermined mapping function. The design space at Level 2 is limited by setting additional design constraints such that the Level 2 design solution behaves similarly to the Level 1 design solution. In this way, design space at Level 2 is limited to include only design points that are known to have desirable performance based the Level 1 design solution. Once a Level 2 design solution is determined, design information from Level 2 is passed to Level 3 using specified mapping functions. This method of passing design information from more simple design levels to more complex design levels is continued until a multilevel design solution is determined at the most complex design level of interest. An interesting trade-off is noticed in the multilevel design template. By limiting design space at complex design levels, there is a risk of excluding the "best" design solution from the feasible search region, therefore selecting a sub-optimum design solution. However, in many cases, fully exploring the design space of a complex design problem is impractical, if not impossible. The design approach presented in the multilevel design template provides a method for limiting the design space based on multilevel system knowledge gained at less complex design levels. Recall from Section 2.2 that the multilevel design template is based on the key concept of robust design, a design approach in which satisficing, rather than optimum, design solutions are selected in order to achieve design performance that is predictable in the presence of uncertainty. Limiting the design space and possibly excluding the optimum design solution from the searchable design region is not a setback for the design approach presented in the multilevel design template. The design approach in the multilevel design template is used to determine satisficing robust design solutions that acceptably achieve design goals, rather than the optimum design solution.

6.1.2 Application of Multilevel Design Template to Blast Resistant Panel Design

The secondary research question and hypothesis are restated from Section 1.3.1:

<u>Secondary Research Question</u>

How can information regarding the robust multilevel design of blast resistant panels by captured and stored in a reusable, computer-executable format?

<u>Secondary Research Hypothesis</u>

By particularizing a generic multilevel design template for the multilevel robust design of blast resistant panels and translating design process information to computer-interpretable modules, information regarding the robust multilevel design of blast resistant panels can be captured and stored in a reusable, computer-executable format.

The second key contribution in this thesis is the application of the multilevel design template to the design of BRPs. With significant design complexity, BRP design is modeled as a multilevel design process. Additionally, uncertainty in BRP loading conditions and material properties is clearly stated in the problem definition such that a multilevel robust design solution is achieved. The multilevel design template is particularized for BRP design by developing computer-executable design modules predicting BRP performance. Following the structure of the multilevel design template, the BRP design problem is divided into three levels of model complexity. BRP performance models are developed for each level and mapping functions are created to exchange information between each level. Based on IDEM, design solutions at each level are robust to uncertainty in material properties, loading conditions, and multilevel process chain. In practice, BRP performance models are developed in MATLAB.

Information is shared among each level such that the robust design solution at one level becomes input information for design at another level. Once BRP performance models and mapping functions are in place, an inductive robust design solution is determined. That is, BRP design decisions are made starting at the least complex level moving towards the most complex level. Information at each stage and in each level of the design process is stored in reusable, modular, computer-executable sub-templates facilitating BRP design space exploration for future design investigation.

6.2 VERIFICATION AND VALIDATION OF MULTILEVEL DESIGN TEMPLATE

The verification and validation of the multilevel design template is examined using the Validation Square construct. The Validation Square is introduced in Chapter 1 as a systematic procedure for building confidence in the validity of design methods. Evidence for method validation is presented at the end of Chapter 3 – Chapter 5 based on theoretical foundations of the multilevel design template and successful application of the template to example problems. In Chapter 6, aspects of method validation presented throughout the thesis are combined to build confidence in the validation of the multilevel design template, shown in Figure 6.2.

The verification and validation of the multilevel design template is established by examined the four quadrants of the Validation Square. A summary of method validation is presented in Table 6.3.

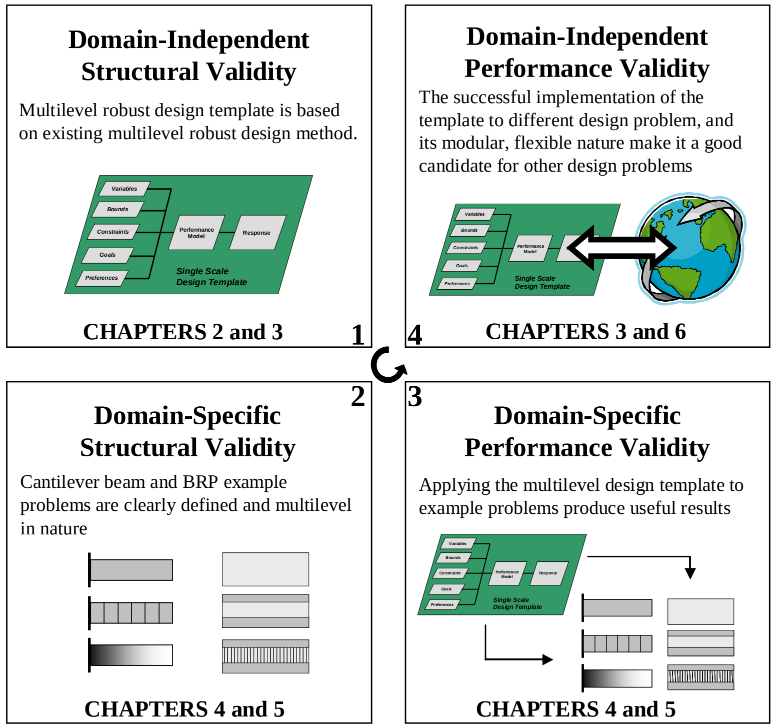


Figure 6.2 – Verification and validation of this thesis using the Validation Square

Table 6.3 – Summary of verification and validation of multilevel design template presented in this thesis

Domain-Independent Structural Validity	
• Critical review of literature that is foundational to the multilevel design template • Topics include multilevel design, robust design, and template-based design • Examination of the way in which foundational topics are combined in multilevel design template	Ch. 2
• Presentation and discussion of an existing multilevel robust design method, IDEM, used as the base method in the multilevel design template • Development of the multilevel design template • Information flow chart illustrates adequate input information for each step in multilevel design template, and adequate output information for subsequent steps	Ch. 3
Domain-Specific Structural Validity	
• Identification of example problems similar to design problems for which the multilevel design template is intended • Example problems include: o Design of a cantilever beam and its associated material o Design of a blast resistant panel • Example problems are multilevel in nature (concurrent product and materials design) • Example problems contain well-defined design variables, constraints, bounds, goals, and uncertainty models • Design solution provide sufficient information for analyzing the effectiveness of the design template	Ch. 4, 5

Table 6.3 (continued) – Summary of verification and validation of multilevel design template presented in this thesis

Domain-Specific Performance Validity	
• Obtained solutions match designer intuition • Validity of numerical solutions tested with a starting point analysis • Internal consistence of cantilever beam and BRP performance models analyzed by varying input information to determine if models behave similar to physical systems	Ch. 4, 5
Domain-Independent Performance Validity	
• Multilevel design template has its foundations in a template-based approach to engineering design • Generic design templates are built for application in a variety of design problems with the expectation of design-template reuse	Ch. 3
• Both example problems presented in this thesis are multilevel in nature • Confidence is built in the success of applying the multilevel design template for all multilevel design problems	Ch. 6

6.2.1 Domain-Independent Structural Validity

Domain-independent structural validity relates to the internal consistency of the design method. Recall from Section 1.4.1 in which several practical steps for asserting domain-independent structural validity are presented. Such validation steps include a thorough literature review of theoretical foundations of the method and an assessment of logical information flow of the design method.

Literature Review of Method Theoretical Foundations

Theoretical foundations of the multilevel design template are presented in Section 2.1 – 2.3 and include a literature review of multilevel design, robust design, and template-based design. Since the multilevel design template is based on well accepted, logical research fields, this builds confidence in the internal consistency of the multilevel design template. The multilevel design template is also based on an existing multilevel robust design method, IDEM (Choi 2005). The internal consistency of IDEM is explored and confirmed in its logical progression of information and successful implementation in example problems (Choi 2005).

Once confidence is built in the structural validity of the theoretical foundations of the multilevel design template, it is important to examine the way in which foundational topics (multilevel design, robust design, and template-based design) are combined. Theoretical foundations *multilevel design* and *robust design* are combined in order to produce a multilevel robust design approach. Combining a class of engineering design problems with a robust design technique is common in engineering design, as demonstrated by the multilevel robust design base method, IDEM (Choi 2005), and this combination preserves the internal consistency of each of the theoretical foundations. Then, *multilevel design* and *robust design* are combined with *template-based design* to develop a design template that supports the robust design of multilevel systems. One of the inherent advantages of a template-based engineering design approach is to map out design information flow in a form that is reusable and archival. That is, adapting an existing design approach to a template-based environment does not affect its internal consistency, it simply arranges and stores design method information in a reusable, modular, and archival form.

Logical Information Flow of Method

An information flow chart in Figure 3.16 and the discussion that follows in Section 3.3.1 builds confidence in the logical progression of thought and information of the multilevel design template. In Section 3.3.1, it is verified that for each step in the multilevel design template there is adequate input information, and that adequate output information is provided for subsequent steps in the design template. Additional confidence is built in the logical information flow of the multilevel design template in the successful implementation of the multilevel design template in two example problems (Chapter 4, Chapter 5). By testing the multilevel design template using example problems, any inconsistent or illogical information exchange would be indicated by an incomplete or unexpected design solution.

6.2.2 Domain-Specific Structural Validity

Domain-specific structural validity is an analysis of the appropriateness of example problems used to test the effectiveness of the method. Example problems should be similar to the problems for which the method was developed, the example problems must represent actual problems that the method would be applied to, and the example problems must produce useful design solutions that can be used to asses the effectiveness of the method.

In this thesis, the multilevel design template is applied to two example problems, the design of a cantilever beam (Chapter 4) and the design of a BRP (Chapter 5). Each problem is clearly defined with known design variables, constraints, goals, and bounds. Each example problem is multilevel in nature demonstrating concurrent product and materials design, and various levels of design models used to predict complex phenomena. Additionally, each example problem contains quantifiable uncertainty. It is asserted that since the example problems are clearly defined, multiscale in nature, and have known uncertainty models, the example problems are similar to problems for which the multilevel robust design method was developed and represent actual problems that the method would be applied to. Appling the multilevel design template to the example problems produces a multilevel design solution that is within specified constraints and bounds. The multilevel design solution provides sufficient data to assess the effectiveness of the multilevel design template based on design variables and design performance. Comments relating to the domain-specific structural validity of the multilevel design template are found in Section 4.3.2 and Section 5.3.1.

6.2.3 Domain-Specific Performance Validity

Domain-specific performance validity is established by applying the method to example problems and testing its effectiveness. Two areas are addressed in order to establish domain-specific performance validity: the usefulness of the numerical design solutions and the overall usefulness of the multilevel design template. These two areas are addressed in Section 6.2.3.

Usefulness of Numerical Results in Cantilever Beam and BRP Example Problems

Establishing the validity of numerical results is necessary to build confidence in the domain-specific performance validity of the multilevel design template. If the numerical results are not valid, then there is no way to assume that the application of the design template to other design problems will produce useful results. In this thesis usefulness of the numerical results are analyzed by conducting starting point analyses and examining the internal consistency of the data. In BRP design Pareto plots and comparing robust vs. non-robust design scenarios are also used to test the domain-specific performance validity of the multilevel design template. In Section 4.3.2 and Section 5.3.2 the domain-specific performance validity is analyzed based on the cantilever beam and BRP design problems.

For the cantilever beam and BRP design problems, starting point analyses are conducted at each design level. For cantilever beam design (Figure 4.3), it is discussed in Section 4.3.2 that various starting points for Level 1 design produce identical results. Cantilever beam design at Level 2 and Level 3 is significantly more complex, and requires the use of the finite element software, COMSOL. Due to the large number of design variables at design Level 2 and Level 3, it is observed that in order to reach logical solutions the starting points must be set to reflect the expected outcome of the design solution. Expected trends in the design solution are left to the discretion of the designer; however, for the cantilever beam example problem, it is fairly obvious that the stronger material should be placed at the base of the beam where there is the greatest stress, and the strength of the material decreases along the length of the beam in the direction of the free end. Therefore, a rigorous starting point analysis is not conducted for cantilever beam design at Level 2 and Level 3. But, the validity of the cantilever beam design solution is established in the internal consistency of the numerical data.

For the BRP design problem (Figure 5.4), starting point analyses are conducted for design at all three levels. For BRP design at Level 3, the starting point analysis revealed one area of instability which is avoided when determining a BRP design solution at Level 3. Starting point analyses at Level 2 and Level 1 indicate stable design solutions for all starting points. Due to its stability based on starting point analyses, the midpoint of design bounds for all design points is used for the starting point in BRP design at all levels. Based on the discussion in Section 5.3.2, it is asserted that the BRP starting point analysis builds confidence in the validity of the numerical BRP design solution.

The second aspect of establishing the validity of the numerical solutions is testing whether changes in input parameters produce results as expected from physical systems. For the cantilever beam design problem, it is observed that increasing the load at the free end increases beam deflection. Similarly, decreasing beam cross-sectional area increases beam deflection, for constant loading conditions. In terms of beam material properties, increasing the strength of the beam decreased beam deflection for constant loading conditions. Based on these observations, it is asserted that the cantilever beam performance models perform similarly to what is expected from a physical cantilever beam under constant loading.

In a similar manner, changes in BRP input parameters are analyzed in order to test the internal consistency of the performance models. Increasing BRP peak pressure load increases BRP back face sheet deflection. Alternatively, increasing characteristic impulse loading time increased back face sheet deflection. In terms of BRP material properties, increasing material strength decreased BRP deflection. These results are expected based on physical systems, therefore, adding value to the internal consistency of the cantilever beam and BRP analysis models.

Additional tests are completed in order to explore BRP design space and add value to the domain-specific performance validity of the multilevel design template based on BRP design. BRP design space is more thoroughly explored using Pareto curves, robust vs. non-robust design scenarios, and inductive vs. deductive solution path analysis discussed in the following paragraphs:

Pareto curves (Section 5.3.2) – Pareto curves are created for BRP design at Level 3 in order to observe the relationship between various design goals while incrementing goal weighing schemes. For BRP design each of the four design goals is compared with all design goals resulting in eight Pareto plots each comparing two design goals. By altering the weighing factors, relationships between the compared goals are identified. In order to more clearly observe trends in goal relationships, an additional graph is displayed for each goal comparison in which outliers are excluded. In Figure 5.17 (a.1), (a.2) comparing mass / area vs. deflection, a slight inverse relationship is detected, best shown in graph (a.2) with outliers removed. That is, as mass / area decreases, panel deflection increases. This trend agrees with designer intuition and is observed in BRP design space exploration in that smaller deflection values can be obtained by increasing the mass / area constraint. Graphs (b.1), (b.2) and (c.1), (c.2) in Figure 5.17 display a comparison of variation in deflection vs. deflection and variation in mass / area vs. mass / area, respectively. For each of these comparisons, a loose direct relationship between BRP performance and BRP variation of performance is observed, illustrating that as performance increases, variation in performance also increases. In graphs (d.1), (d.2), (e.1), (e.2), (f.1), and (f.2) in Figure 5.17 no conclusive relationships are detected among the compared quantities of variation in mass / area vs. deflection, variation in deflection vs. mass / area, and variation in mass / area vs. variation in deflection.

Outlier points in BRP Level 3 Pareto graphs are analyzed in order to determine the "best" weighting scheme to achieve design goals. BRP performance is compared using the determined "best" weighing scheme and an equal weighting scheme. Surprisingly, the "best" BRP weighting scheme places the greatest emphasis on maximizing robustness with respect to variation in BRP deflection. The goal of minimizing BRP deflection is weighted at zero. The two remaining goals are weighted similarly. A comparison reveals similar BRP performance for both weighting schemes. However, for the "best" weighting scenario the deflection goal is not considered in reaching a design solution, leading to the reasonable conclusion that the deflection goal has little effect in achieving BRP performance and robustness goals. Also, it is observed that the robustness goal related to variation in deflection is significant in achieving design solutions the best meet design goals. Such conclusions, which are beyond designer intuition, could be useful starting points when developing weighting schemes for BRP design at increased levels of model complexity.

Robust vs. non-robust design scenarios (Section 5.2.3) – Two design scenarios are investigated in order to assess the robustness of the multilevel design solution. In the robust-design scenario, each of the four design goals is weighted equally. In the non-robust design scenario, the two performance goals are weighted equally, and the robustness goals are removed from the design process. The robust and non-robust design scenarios are maintained throughout all levels of the multilevel design process and results are analyzed. Robustness of design solutions is measured using the HD-EMI robustness metric. It is found that early in the BRP design process (Level 1) design solutions obtained from a robust goal weighting scheme are more robust than design solutions achieved based on non-robust goal weighting. As the designer progresses through the design process (Level 2), the robust and non-robust design solutions begin to have similar levels or robustness. Then, late in the design process (Level 3) design solutions from a non-robust weighting scheme are actually more robust than design solutions from a robust weighing scheme. This unexpected trend in multilevel design is attributed to the mapping functions which control what information is passed to subsequent design decisions and set the tone for design solutions later in the design process.

Inductive vs. deductive solution path (Section 5.2.3) – Due to its relative simplicity, the BRP design problem can be solved directly without the use of the inductive approach embedded in the multilevel design template. The direct (or deductive) design solution is obtained and compared with the inductive BRP design solution determined using the multilevel design template. In general, BRP design goals are more closely achieved when using an inductive design approach. However, BRP performance for an inductive design approach and direct calculation are similar. Since each design scenario produces similar BRP performance it is concluded that differences in design variable data indicate different by comparable solutions in BRP design space. The inductive vs. deductive design comparison indicates that while implementing an inductive and deductive solution path in BRP design produces different design solutions, the design solutions measure remarkably similar performance. This realization adds value to the inductive design approach. In complex BRP design, an inductive design approach simplifies the design process by limiting the design space at complex design levels. Based on the information in the previous section, it is observed that an inductive solution path in which complex design space is limited does not inversely affect the overall performance of the attained BRP design solution.

Usefulness of Multilevel Design Template in Solving Example Problems

A second aspect of establishing the domain-specific performance validity of the multilevel design template is in analyzing its overall usefulness by addressing the question, “Are there any advantages to using the multilevel design template compared to a traditional product design process?” In Section 4.3.2 and Section 5.3.2 the advantages of implementing the multilevel design template in cantilever beam and BRP design are discussed and include: the partitioning of a complex design problem to levels of design complexity, the support of agile design space exploration via performance models and mapping functions, and the collection and organization of design information for use in future design analysis. In summary, the structure of the multilevel design template is useful in guiding a designer in information collection and allocation, design space exploration, and solution analysis of a complex, multilevel design problem.

6.2.4 Domain-Independent Performance Validity

Domain-independent performance validity is asserted by showing that the method is useful beyond the selected example problems. This step in the Validation Square is often referred to a “leap of faith” in which the designer must speculate as to the future success of the method based on confidence built from the previous three quadrants of the Validation Square. The domain-independent performance validity of the multilevel design template is established in the successful application of the template in two multilevel example problems, and the mutable nature of the design template. Due to the successful application of the design template to the two multilevel example problems, it is inferred that the template will produce equally beneficial results when applied to additional multilevel design problems. Also, the design template is problem-generic, modular, and mutable—all characteristics that allow it to be adapted to suit future design problems. Recall from Chapter 2 that one of the key requirements of a design template is reusability. By nature, design templates are designed for application in a variety of design problems. Therefore, it is asserted with confidence that the multilevel design template can be applied to additional multilevel design problems producing beneficial design solutions.

Additionally, the domain-independent performance validity quadrant of the Validation Square involves investigation of the underlying theoretical contributions in the multilevel design template. These theoretical contributions describe the essence of the multilevel design template, and are advances in engineering design that can be applied to design problems beyond those investigated in this thesis. The intellectual contributions of the multilevel design template that can be applied to future design problems include: the application of an existing design method to a template-based design environment, the partitioning of complex design problems based on level of model complexity, and the limitation of vast design space by identifying regions likely containing satisficing design solutions. First, the multilevel design template is developed based on two existing design methods, cDSP and IDEM. The overall structure and information flow of these two methods are combined and extended to a template-based environment for multilevel robust design. The multilevel design template is then extended to computer-executable modules capable of capturing and directing design information for specific design problems. The concept of adapting an existing design method to a template-based computer executable environment can be applied to a variety of design problems outside of the domains investigated in this thesis. Next, the multilevel design template introduces a new method for dividing multilevel design problems based on levels of model complexity. A complex design problem can be divided in a multilevel-multiscale manner (based on changes in length and / or time measure) or in a multilevel-homogenization manner (based on averaging or homogenization techniques to achieve model simplicity / complexity). This underlying concept of the multilevel design template can be applied as a method for partitioning complex engineering design problems beyond the examples investigated in this thesis. Finally, the multilevel design template provides an approach for achieving satisficing design solutions to complex design problems. Based on a series of increasingly complex design decisions connect by mapping functions, design information from simple design levels is transferred to complex design levels in order to limit design space and identify regions likely containing satisficing design solutions. This approach for simplifying prohibitively complex design problems represents one of the key contributions in the multilevel design template, a concept which can be applied to a variety of complex design problems, regardless of domain. The three key theoretical contributions listed in the previous paragraph detail aspects of the multilevel design template that can be applied to many complex design problems, building confidence in the domain-independent performance validity of the multilevel design template.

6.3 ADDRESSING THE RESEARCH GAP – COMPLETED AND FUTURE WORK IN TEMPLATE-BASED MULTILEVEL DESIGN

In Section 6.3, an assessment of how well the identified research gap has been addressed and opportunities for advancement relating to the multilevel design template and example problems are presented. The research gap is revisited in Section 6.3.1 with a discussion of how well the identified gap has been addressed. In Section 6.3.2 future work relating to the multilevel design template is presented. Section 6.3.3 contains opportunities for improvement relating to the example problems, cantilever beam design and BRP design. Section 6.3.4 contains speculation as to how template-based multilevel robust design can be extended for engineering design of the future.

6.3.1 Addressing the Research Gap

In Section 2.4, two key research gaps in multilevel template-based design are identified. The extent to which these gaps have been filled in this thesis is addressed in Section 6.3.1. Figure 6.3 lists the key gaps investigated in this thesis including: the extension of multiscale design concepts from the materials design community to encompass a multilevel product and materials design approach, and the combination of existing design methods with a template-based design approach. Recall from Section 2.4, that research in multilevel template-based design is aimed at the development of a detailed multilevel design method composed of reusable design templates that addresses the critical needs of a multilevel design process (see Section 1.1.2, Section 2.1.2 and Table 2.2 for a list of critical needs). Addressing the research gaps shown in Figure 6.3 is a small step in achieving the overall goal of a detailed multilevel design method.

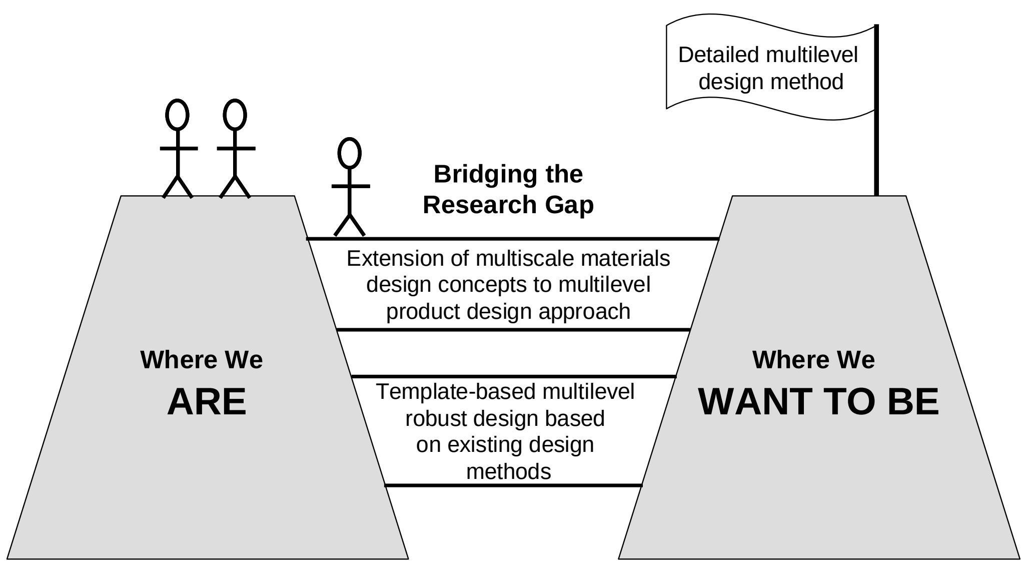


Figure 6.3 – Addressing the identified research gap

In the engineering design community there are a variety of detail design methods for single scale design (e.g., Pahl and Beitz 1996). One of the main research goals in the field of multiscale (or multilevel) design methodology is the development of a detail, systematic design method to characterize multilevel systems and address the critical needs of a multilevel design process (see Section 1.1.2, Section 2.1.2 and Table 2.2 for a list of critical needs). A detail multilevel design method would provide the procedural steps needed for multilevel design with sufficient generality for application in a variety of design problems. The multilevel design template presented in this thesis in response to the identified research gaps is one step in the journey to creating a multilevel design method by providing a framework for multilevel design. However, the multilevel design template lacks the details needed for a systematic multilevel design method. Researchers must gain a greater understanding of the elaborate interaction patterns of various levels in complex design problems before a detail design method for multilevel design is developed. The work presented in this thesis is intended to help close this wide research gap by providing design tools in template-based multilevel robust design that can be built on in future research. A discussion of the extent to which the identified research gaps have been addressed in this thesis follows.

Addressing the Research Gap Relating to Multilevel Robust Design – From Multiscale Design to Multilevel Design

In Section 2.4.1 a research gap is identified based on the extension of multiscale design concepts from the materials design community to a multilevel product and material design approach. It is observed that the multiscale modeling and design approach used in materials design based on dividing a design problem according to length and / or time scales can be abstracted to include a wider range of engineering design problems. This research gap is addressed in this thesis with the distinction of multilevel design (different from multiscale design) as a design problem which is modeled and analyzed at various levels of model complexity or resolution. The complexity or resolution of a design level is based on the number of degrees of freedom at that level. Once a multilevel design problem is identified, design levels are defined and performance models are created at each level. Information at each level is combined to determine the multilevel design solution.

Addressing the Research Gap Relating to a Template-Based Approach to Multilevel Robust Design

It is also identified that a template-based approach to multilevel design problem currently does not exist in the engineering design community. This research gap is addressed in this thesis with the development of a multilevel design template based on a multi-objective design template, the cDSP, and a multilevel robust design method, IDEM (see Section 3.1, Section 3.2). The multilevel design template presented in this thesis provides the framework and information flow necessary to frame and solve a multilevel design problem. Additionally, multilevel robust design goals are infused in the multilevel template such the multilevel design solution is robust to propagated process chain uncertainty. The next phase in addressing this research gap is to apply the multilevel design template to multilevel example problems (see Figure 3.13, Figure 3.15). Two example problems are solved (Chapter 4, Chapter 5) as part of the verification and validation of the multilevel design template. The multilevel design template can be added to as future research leads to a detailed multilevel design method based on design templates.

6.3.2 Future Work Related to Multilevel Design Template

In Section 6.3.1 areas for future improvement relating to the multilevel design template are discussed. To begin, the knowledge gap that arises from particularizing the generic multilevel design template for a specific multilevel design problem is addressed. Then, it is shown how dividing a design problem according to levels of model complexity can be extended for future design processes. Also, current limitations in the method for achieving robust design solutions are addressed.

The first area for future advancement of the multilevel design template is the development of guidelines for the particularization and application of the multilevel design template in specific design problems. The multilevel design template presented in Chapter 3 is a generic design structure used as a guide in multilevel design process formulation and design-making. In order for the multilevel design template to be useful, it must be particularized for a specific multilevel design problem. In this thesis, it is left to the discretion of the designer to determine how the multilevel design template should be altered for application in a specific design problem. It would be useful to develop guidelines for altering the multilevel design template such that the augmented multilevel design template performs as expected. Additionally, in this thesis, it remains unclear how to translate the structure of the multilevel design template into computer-executable system response model sub-templates. Currently there are no guidelines for how information is shared among computer executable sub-templates. For the example problems examined in this thesis, design information is shared among design levels by allowing a robust design solution at one scale to become input information to determine a robust design solution at another scale. However, in this thesis, no standard is determined for sharing design information in a computational template-based design environment. For future work, it is beneficial to research methods for applying generic design tools, such as the multilevel design template, to the computational design environment of specific example problems.

The next area for advancing research of template-based multilevel design involves further examination of design complexity as a method for partitioning multilevel design problems. In this thesis, multilevel design problems are divided based on model precision, or model complexity. It is observed that simply increasing the complexity of a prediction model does not necessarily increase its precision (in this thesis, it is assumed that an increase in complexity is synonymous with an increase in model precision).

Future work in this area should clarify differences in model complexity and model precision. Additionally, in this thesis, the measure for design complexity is the number of design variables used in system prediction models (as the number of design variables increases, model complexity increases). However, measuring design complexity based solely on the number of design variables is perhaps too simplistic. Complexity in a design problem is defined by many factors including the number of design variables, couplings between design variables, and the influence of each design variable on system performance. In future work, it would be beneficial to thoroughly investigate what is meant by the precision of a system prediction model and develop a method for measuring system complexity based on a variety of factors. The concepts of model complexity presented in this thesis open the door to a new method of model classification that has many areas for research opportunities.

An additional area of future work relating to the multilevel design template involves selecting the best strategy for incorporating robust design concepts in the multilevel design template. In this thesis, multilevel robust design is achieved by setting a design goal to maximize system robustness at each design level (based on the robustness metric, HD-EMI). A research opportunity exists in expanding this approach to include section of the best method for achieving multilevel robust solutions. Ideally, a “multilevel robust method library” containing many approaches for achieving multilevel robust solutions would exist such that a designer could select the robustness strategy that best suits a particular multilevel design problem. In future research it would be interesting to compare various multilevel robust design methods incorporated in a template-based multilevel design environment, highlighting the advantages and disadvantages of each approach.

6.3.3 Future Work Related to Example Problems

In Section 6.3.2 research opportunities relating to the example problems completed in this thesis are discussed. Since each example problem involves the concurrent design of product and material, a research opportunity exists in increasing the precision at which system prediction models are developed to incorporate more detailed material performance models. For the examples in this thesis, material behavior models operate at the continuum level by predicting system performance based on homogeneous material properties. In order to fully explore the benefits of concurrent product and materials design in the examples in this thesis, material behavior models at the meso-, micro-, nano-, etc. levels should be investigated. With current computation power, developing micro- and nano- material models and making design decisions at these scales requires significant computation cost. However, less expensive steps, such as modeling material at failure sites with more precise models using FEA software, could be taken to improve the quality of concurrent product and materials design solutions.

Future Work – Cantilever Beam Example Problem

The cantilever beam example problem is used to illustrate the advantages of a template-based approach to multilevel design in a concurrent product and material design process. Design freedom exists in the cross-sectional area of the cantilever beam and its material properties. Areas for future research include extending the design freedom of the cantilever beam to include variation in material properties in more than one direction and dynamic cross-section geometry. For this thesis, the material properties of the cantilever beam vary in one direction, along the length of the beam (x-direction). For a more realistic materials design illustration, the material properties of the beam should be alloyed to vary in all three directions of the beam (x-, y-, and z-direction). This increase in material freedom would result in a more accurate picture of the ideal material for cantilever beam design. For cantilever beam design in this thesis beam cross-section geometry remains constant. For future work, it would be interesting to allow beam geometry to vary and investigate cross-sectional geometry trends in cantilever beam design. Expanding design freedom in material properties and beam cross-section would produce a more complex design environment for concurrent product and materials design.

Future Work – BRP Example Problem

There are four main areas for future work in BRP design including: model verification using FEA, loading analysis and verification, core layer design and analysis, and identification of key working principles in BRP design. Each of these research opportunities in BRP design is discussed in the following paragraphs. In addition to these areas for future work, a broader area for reach opportunity includes increasing the detail of BRP material models, discussed in Section 6.3.2, first paragraph. By design BRP material with increased precision the advantages of concurrent product and material design are more easily observed.

Current BRP multilevel performance models are based on the work of Hutchinson and Xue, 2005 and implemented in MATLAB. These models represent an approximation of BRP performance and should be verified using FEA. Jin Song and Gautam Puri, former and current undergraduate researchers of the Systems Realization Lab at Georgia Tech, initiated the verification of BRP analytical models using FEA software ABAQUS. This work is still in progress, although preliminary solutions can be found in Appendix D. Additionally, a BRP loading analysis should be conducted in order to verify approximations in BRP loading conditions. In practical use, BRPs are designed to protect against blast loading, similar to what is found in a wartime environment. Recall from Section 5.1.1 that BRP impulse loading (blast loading) in BRP prediction models is modeled as a pressure pulse, with constant pressure over the panel top face sheet, varying in time. For future work, FEA should be used to determine if the constant pressure blast wave is an acceptable approximation for BRP impulse loading conditions.

The key feature of a BRP is its honeycomb core layer which dissipates energy due to core crushing. There are many research opportunities relating to altering the honeycomb core layer in order to achieve greater energy dissipation while decreasing core mass. In this thesis, BRPs with square honeycomb cores are considered. However, research should be conducted to determine the best topology for the BRP core layer in order to maximize energy absorption while minimizing BRP mass. Future work in BRP design includes concurrent BRP product, material, and core topology design. Another promising research opportunity in BRP design is investigating the advantages of placing a fill material in the cells of honeycomb core to increase energy dissipation. A lightweight granular fill material placed in BRP honeycomb cells would dissipate blast energy due to friction among fill material particles without significantly increasing BRP mass. Design variables in a BRP with fill material could include the properties of the fill material (grain density, grain size, grain roughness, etc.), location of fill material in BRP core, and amount of fill material in the filled cells.

Finally, for future work in BRP design, it is important to identify the key working principles of BRP energy dissipation. The basic mechanism for energy dissipation in the BRP design investigated in this thesis is known to be crushing of the core layer. However, it is important investigate other methods for dissipating energy and minimizing deflection that could be incorporated in BRP design. For future BRP design, the role of each layer and its features (material properties, topology, layer thickness, etc.) in minimizing back face sheet deflection should be fully understood and modeled. By doing so an inductive and systematic design method for BRP design which meets specific goals and requirements could be accomplished.

6.3.4 Extending Template-Based Multilevel Robust Design Methodology

In Section 6.3.3 future work relating to the general study of template-based multilevel robust design is investigated. The multilevel design template presented in this thesis is only one contribution to the vast research field of engineering design, and specifically, template-based design. Based on knowledge gained in developing and implementing the multilevel design template in this thesis key areas for advancement in template-based multilevel design include: the development of a multidimensional multilevel design template and the development of a template-based approach to a detailed multilevel design method. These areas for research opportunity are discussed in Section 6.3.3.

Template-Based Multidimensional Multilevel Design

The multilevel design template in this thesis is useful for design problems divided according to levels of model precision or complexity. However, there are many other factors that can be used to divide a complex design problem such as component length scale, system reaction time, groups relating to similar system behavior, and components along a single flow of information. An inherent weakness in the multilevel design template is that it is only capable of managing design problems divided using a single metric. An exceptionally complex design problem may be best described using multiple design metrics to define design levels, referred to as multidimensional multilevel design. An example of a multidimensional multilevel design problem is a materials design problem divided according to component length scale and system reaction time. Developing a framework for multidimensional multilevel design is currently unexplored in the engineering design community and is left as future work.

6.3.5 Vision for Template-Based Engineering Design of the Future

The main focus in this thesis is the adaptation of design methods into design templates, thus increasing the reusability, modularity, flexibility, and archival nature of the design method. While a template-based approach to engineering design is beneficial for today, what role will design templates play in design processes of the future? Due to the distributive, collaborative, and increasingly complex nature of engineering design of the future, we assert that design templates will grow in popularity and necessity as design process building blocks for engineering design of the future.

It seems inarguable that society is moving towards a distributive, collaborative, and increasingly competitive social and business environment. It is already observed that as engineering design problems increase in complexity, design experts from around the world focus united effort to solve a single design problem. We speculate that this trend will continue to increase, resulting in the need for collaboration in almost all design problems. There is the additional pressure of quickly and constantly designing new products in keeping with ever-changing consumer needs. Design collaborating is also beneficial in reducing design process time, thus reducing a product's time to market. Recall the information flow of the multilevel design template in Figure 3.16. As illustrated, the multilevel design process is carried out by a single designer. In actuality, this rarely occurs. Today, and certainly in the future, design process information flow contains a variety of experts making decisions at various points along the design process, as illustrated in Figure 6.4

As shown in Figure 6.4, information flow in a collaborative design process contains many individuals supplying design expertise at various points in the design process. The design is coordinated by a single designer or group of designers who has a working knowledge of the overall design process without full understanding of each design decision. This model of design collaboration is expected as engineering design problems of the future continue to increase in complexity.

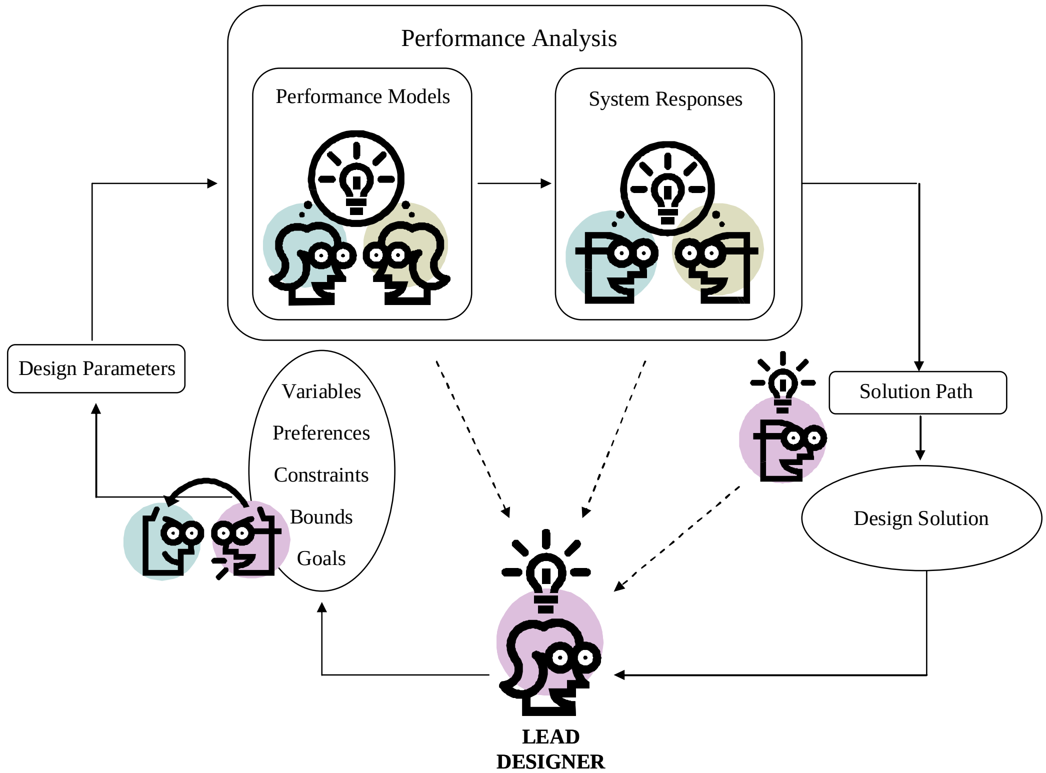


Figure 6.4 – Collaborative, distributive multilevel design process

In light of the distributive and collaborative nature of engineering design in the future, how can design templates be used to add value and efficiency to engineering design? By observing the key characteristics of design templates, we assert that a template-based approach to engineering design is a valuable and necessary step in design process design of the future. Key characteristics of design template are listed below:

Reusable – Design templates are reusable, meaning that time and money are saved by implementing the same design template in multiple design problems.

Modular – Design templates can be used as building blocks to tailor the design process of a particular application. Additionally, components within the design template can be altered or replaced to meet design process requirements without affecting the remainder of the template. The modularity of design templates allows for quick and easy design process reconfiguration to accommodate for changes in the design problem.

Mutable – Design templates are capable of modification in order to meet the needs of a specific design process, or aspect of a design process implying that a similar design template can be applied to a variety of design problems with only slight modification.

Archival – Design templates provide the construct to store and transport information supporting a collaborative and distributive design environment.

Our vision for engineering design of the future includes the wide-spread use of design templates as building blocks in design process design. As design templates increase in popularity, a design template data base may be created. The design template data base contains a searchable collection of design templates that can be combined to create personalized design processes. We envision a future in which, once a design process is defined, a designer's first action will be to build a suitable design process from a design template database. What steps must be taken in the design community to make this a reality? We assert that the development of design templates based on existing and new design processes is necessary and crucial to encourage efficient collaborative and distributive design of complex systems of the future.

6.3.6 Lessons Learned from the Design and Application of a Multilevel Design Template

In creating the multilevel design template presented in this thesis and applying it to cantilever beam and BRP example problems, I have gained valuable insight into the benefits and challenges of a template-based approach to multilevel design. In Section 6.3.6, I present key lessons learned based on the ideas and information presented in this thesis. Lessons learned are discussed below by addressing three questions that I often pondered during the creation and application of the multilevel design template.

Key Lesson #1: Why is a multilevel design approach beneficial for solving complex design problems?

Multilevel design is a strategy for dividing complex design problems into levels of manageable complexity. In engineering design, a trend of increasing product complexity is observed. In many cases, design problems are becoming so complex that modern computational tools are not sufficient to produce satisfactory design solutions within design process time and cost constraints. By implementing a multilevel design approach, complex design problems are analyzed at various levels of model complexity. Models at low levels of complexity are used for extensive design space exploration. Design knowledge gained from investigating less complex models is mapped into more complex design models thereby reducing the amount of design space exploration necessary at complex levels. A multilevel design approach is intended to produce satisfactory design solutions to complex design problems at reduced time and computation cost compared to traditional single-level design processes. I have observed that one of the key challenges in multilevel design is the translation of design information among various design levels such that design knowledge is preserved without over-complicating future design decisions.

Key Lesson #2: Is my design problem multilevel? How do I define levels?

After careful consideration, I conclude that all design problems are multilevel in nature. All design problems can be modeled at increasing or decreasing levels of model complexity. Consider a simple design problem that may not seem multilevel at first glance—the design of a cantilever beam. However, by including all aspects of cantilever beam design (product and material) the multilevel nature of cantilever beam design is soon realized. There are no regulations for defining design levels in a multilevel design problem. However, after completing two multilevel example problems, I have observed several guidelines indicating a change in design level including: a change in computation tools used to describe product performance, a change in length and / or reaction time in modeling product performance, and a change in fundamental phenomena used to describe produce performance. It is left to the informed designer to define the “best” number of design levels and location of design levels in a multilevel design process. Defining too many design levels may over-complicate design decisions and introduce unnecessary propagated process chain uncertainty. Defining too few design levels may result in the failure to capture relevant phenomena in a multilevel design problem.

Key Lesson #3: Mapping functions in a multilevel design template, friend or foe?

The key to a successful multilevel design template is in the careful design of its mapping functions. Mapping functions are information highways used to transfer design information among design levels. Specifically, mapping functions are created such that design knowledge obtained at less complex design levels can be transferred to more complex deigns levels. A particular multilevel design problem does not have a unique set of mapping functions. Rather, it is the role of the designer to create mapping functions that best transfer design information in order to meet design objectives. Mapping functions are powerful tools in multilevel design, setting the tone for design designs at more complex levels of design. A poor selection of mapping functions may result in an over-limited design space for which a satisfactory design solution cannot be found. Inappropriate mapping functions are also capable of transferring incorrect or irrelevant design information, resulting in an undesired multilevel design solution. But, with well-crafted mapping functions, the essence of design knowledge is passed throughout a multilevel design process enhancing design decisions at later and more complex stages of design.

6.4 SYNOPSIS OF CHAPTER 6

Central topics in Chapter 6 include research contributions of this thesis, verification and validation of the multilevel design template, and opportunities for improvement relating to the multilevel design template and example problems. The main research contributions of this thesis include the development of a multilevel design template and the application of the design template to the multilevel design of BRPs. Verification and validation of the multilevel design template is established based on the Validation Square construct. Aspects of method validation presented throughout this thesis are brought together in Section 6.2 where the verification and validation of the multilevel design template is established. Chapter 6 continues with future work relating to the multilevel design template, the example problems presented in this thesis, and the advancement of template-based multilevel design methodology for complex design problems of the future. At the conclusion of Chapter 6 lessons learned from the design and application of a multilevel design template are discussed.

APPENDIX A

PAHL AND BEITZ SYSTEMATIC PRODUCT DESIGN METHOD

The flow of information in the systematic design method developed by Pahl and Beitz is shown in Figure A.1 (Pahl and Beitz 2006).

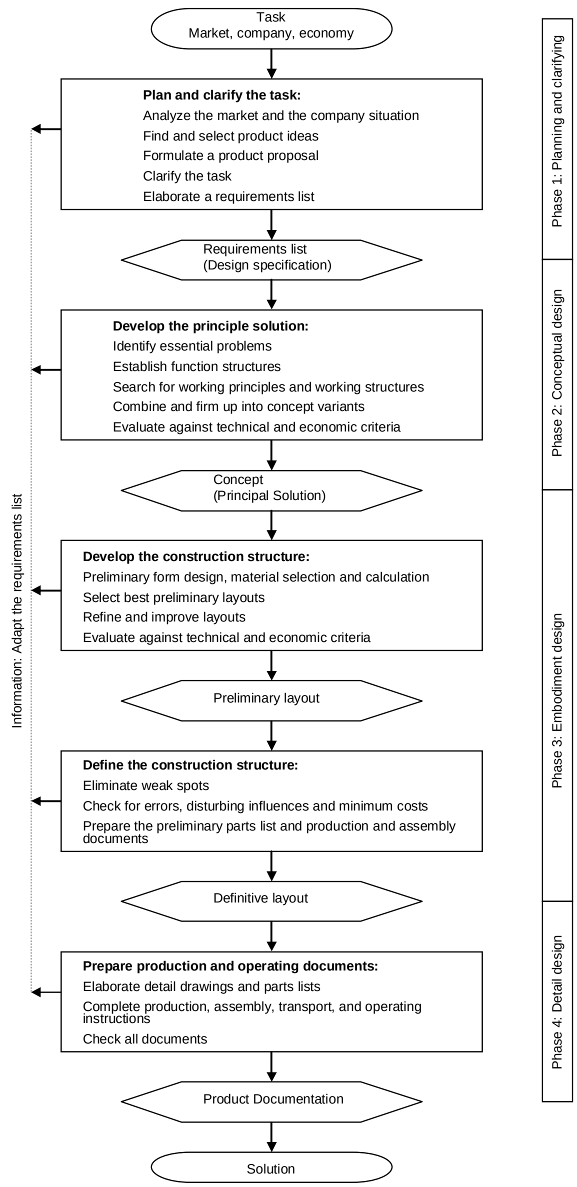


Figure A.1 – Pahl and Beitz systematic product design method (Pahl and Beitz 2006)

APPENDIX B

REQUIREMENTS LISTS FOR BLAST RESISTANT PANEL DESIGN

Requirements lists for the design of a blast resistant panel are presented in Table B.1 – Table B.3. The requirements list presented below relate to the motivation for completing the BRP design example problem. The motivation is categorized as: advancement of multiscale robust design methodology, verification and validation of proposed template based approach to multiscale robust design, and to satisfy BRP design requirements developed by the customer.

Table B.1 – BRP Requirements List for the advancement of multiscale robust design methodology

Request:	**Title:**	**Originator:**	**Issued on:**
BRP	**Blast Resistant Panel (BRP)**	Hannah Muchnick, Matthias Messer, Stephanie Thompson	24-Jul-06

Problem Statement: A blast resistant panel (BRP) that provides increased energy absorption per unit mass compared to conventional solid plates is to be designed. Given the design goals of minimizing deflection for a given mass constraint, a material can be designed to meet the BRP performance requirements. Since there is uncertainty associated with the manner in which the BRP will be loaded and the BRP material properties, a BRP that is robust to uncertainty in loading conditions and material properties is designed.
Principal solution: Multilayer sandwich structure with extruded honeycomb core(s):
Motivation: Advancement of multiscale robust design methodology (including robust materials design)

Type		**D W**	**Requirements**		**Data and Comments**	
			No.	**Description**		
Material	**Material Design "Multilayer Sandwich Structure with Extrudable Core(s)"**			Material composition	Sandwich-structure consisting of bondable layers	
				Functionality	Energy absorption	
					Structural stability	
				Material-design goals	Maximize energy absorption due to core crushing and friction between fill material particles	
					Minimize variation of energy absorption	
					Dissipate energy in multiple directions	
					Transfer vertical load into shear direction	
				Material-design variables	Design variables	Yield strength
						Density
						Microstructure
					Number of layers	
					Layer thickness(es)	h_f
					Core topology	Square

Table B.1 (continued) – BRP Requirements List for the advancement of multiscale robust design methodology

					Triangle
					Hexagon
					Chiral
				Core dimensions	H, B, h_c
				Relative density (R_c)	Core material / total volume of core >= 0.07
			Material-design constraints	Mass/area constraint	
				Deflection constraint	
				Shear-off constraint	
			Material-design bounds	Material properties	Density: 1000 - 10000 kg/m^3
					Yield strength: 100 - 1500 Mpa
				Layer thickness	Face sheet (h_f): 1 - 2.5 mm
					Core (H): 5 - 50 mm
				Core topology	Cell spacing (B): 1 - 20 mm
					Cell wall thickness (h_c): 0.1 - 5 mm
			Material-design simplifying assumptions	No strain hardening	
			Physical properties	Low aging	
			Chemical properties	Low oxidation	
			Failure modes to be avoided:	No buckling:	R_c >= 0.07
				No delamination (shearing of face sheets)	
				No rupture	
				No structural collapse	
Design Scales			Material-design length scales	Macro-scale	
				Meso-scale	
				Micro-scale	
			Material-design time scales	Product reaction time	0.0001 sec
				Product life cycle time	10 years
				Product evolution time	100 years
Robust			Robust to	Uncertainty in loading conditions (impulse shape and direction)	
				Uncertainty in material properties	
				Uncertainty in design factors	
				Uncertainty in simulation models	

Table B.2 – BRP Requirements List for the verification and validation of template based approach to the robust design of multiscale systems

Request:	**Title:**	**Originator:**	**Issued on:**
BRP	**Blast Resistant Panel (BRP)**	Hannah Muchnick, Matthias Messer, Stephanie Thompson	24-Jul-06
Problem Statement: A blast resistant panel (BRP) that provides increased energy absorption per unit mass compared to conventional solid plates is to be designed. Given the design goals of minimizing deflection for a given mass constraint, a material can be designed to meet the BRP performance requirements. Since there is uncertainty associated with the manner in which the BRP will be loaded and the BRP material properties, a BRP that is robust to uncertainty in loading conditions and material properties is designed. The BRP design problem is used to demonstrate and validate a multiscale robust design process.			

Table B.2 (continued) – BRP Requirements List for the verification and validation of template based approach to the robust design of multiscale systems

Principal solution:
Multilayer sandwich structure with extruded honeycomb core(s):

Motivation:
Multiscale design problem used to demonstrate and validate a template based approach to multiscale robust design as presented in a MS Thesis.

Type	D W	Requirements No.	Requirements Description	Data and Comments	
Material; Material Design "Multilayer Sandwich Structure with Extrudable Core(s)"			Material composition	Sandwich-structure consisting of bondable layers	
			Functionality	Energy absorption	
				Structural stability	
			Material-design goals	Maximize energy absorption due to core crushing and friction between fill material particles	
				Minimize variation of energy absorption	
				Dissipate energy in multiple directions	
				Transfer vertical load into shear direction	
			Material-design variables	Design variables	Yield strength
					Density
					Microstructure
				Number of layers	
				Layer thickness(es)	h_f
				Core topology	Square
					Triangle
					Hexagon
					Chiral
				Core dimensions	H, B, h_c
				Relative density (R_c)	Core material / total volume of core >= 0.07
			Material-design constraints	Mass/area constraint	
				Deflection constraint	
				Shear-off constraint	
			Material-design bounds	Material properties	Density: 1000 - 10000 kg/m^3
					Yield strength: 100 MPa - 1500 MPa
				Layer thickness	Face sheet (h_f): 1 - 2.5 mm
					Core (H): 5 - 50 mm
				Core topology	Cell spacing (B): 1 - 20 mm
					Cell wall thickness (h_c): 0.1 - 5 mm
			Material-design simplifying assumptions	No strain hardening	
			Physical properties	Low aging	
			Chemical properties	Low oxidation	
			Failure modes to be avoided:	No buckling:	R_c >= 0.07
				No delamination (shearing of face sheets)	
				No rupture	
				No structural collapse	
Design Scales			Material-design length scales	Macro-scale	
				Meso-scale	
				Micro-scale	

Robust			Material-design time scales	Product reaction time	0.0001 sec
				Product life cycle time	10 years
				Product evolution time	100 years
			Robust to	Uncertainty in loading conditions (impulse shape and direction)	
				Uncertainty in material properties	
				Uncertainty in design factors	
				Uncertainty in simulation models	
MS Thesis			Complete mathematical models	8/1/2006	
			Response surface model	8/10/2006	
			Implement IDEM method to BRP design	9/1/2006	
			Complete BRP robust design solution	9/30/2006	
			Write BRP chapter of thesis	10/1/2006	
			MS thesis presentation	10/18/2006	
			Necessary revisions to BRP solution	11/15/2006	

Table B.3 – BRP Requirements List for satisfying customer requirements

Request:	**Title:**	**Originators:**	**Issued on:**
BRP	**Blast Resistant Panel (BRP)**	Hannah Muchnick, Matthias Messer, Stephanie Thompson	24-Jul-06

Problem Statement:

A blast resistant panel (BRP) that provides increased energy absorption per unit mass compared to conventional solid plates is to be designed. In one application of this design, BRPs can be attached on the outside of military vehicles to protect them from explosions. The BRP must be robust to changes in loading conditions. Since there is uncertainty associated with the manner in which the BRP will be loaded, a BRP that has a relatively consistent performance in a changing environment has to be designed.

Principal solution:

Multilayer sandwich structure with extruded honeycomb core(s):

Motivation:

Satisfaction of customer requirements initiated by the Army Research Lab (ARL)

Type		**D W**	**Requirements**		**Data and Comments**	
			No.	**Description**		
Engineering System "Blast Resistant Panel"	Geometry			Size:	Length	1 m
					Breadth	1 m
				Arrangement	Multilayer sandwich panel with extruded honeycomb core(s)	
				Number of layers	>= 2	
				Boundary conditions	To be determined for FEM simulations	
				Shape	Overall panel is flat square	
					Core(s) have various topologies	
				Interfaces	Interfacing with flat and curved surfaces	
				Mutability	Mutable core(s)	
				Modularity	Modular layers	
				Scalability	Scalable in size (in length and breadth direction)	
	Forces			Load	Impulse in air uniformly distributed over front face sheet	
				Magnitude of force	Blast pressure: 19000 (190 MPa) - 28000 N/s (280 MPa)	
				Direction of force	Blast encounters front face sheet at a varying angle of incidence	
				Weight (mass/area)	<= 25 kg/m^2	
				Deformation	<= 0.01 m (10% of length)	
				Frequency	BRP intended for one time use	

Table B.3 (continued) – BRP Requirements List for satisfying customer requirements

	Energy			Input	Kinetic energy	
				Output	Thermal energy	
				Efficiency	Low => high energy absorption	
				Loss	High energy dissipation through core crushing, face sheet bending and face sheet stretching and friction between fill material particles	
				Temperature	- 10 degrees Celsius (winter) to + 50 degrees Celsius (summer)	
				Cooling	Air cooling (natural convection)	
Material	Material Design			Material composition	Sandwich-structure consisting of bondable layers	
				Functionality	Energy absorption	
					Structural stability	
				Robust to	Uncertainty in loading conditions	
					Uncertainty in noise factors	
					Uncertainty in design factors	
					Uncertainty in simulation models	
	Properties			Physical properties	Low aging	
				Chemical properties	Low oxidation	
				Failure modes to be avoided:	No buckling:	$R_c >= 0.07$
					No delamination (shearing of face sheets)	
					No rupture	
					No structural collapse	
Meetings with ARL				Initial proposal	Fall 2005	
				Initial presentation	Fall 2005	
				CCMD Meeting at Georgia Tech - 2006	2/27/2006 - 2/28/2006	
				Quad chart	Spring 2006	
				Meeting at ARL	6/26/2006	
				CCMD Meeting at Penn State - 2006	8/25/2006 - 8/26/2006	
				CCMD Meeting at Georgia Tech - 2007	Winter 2007	
				Paper on robust design templates	Spring 2007	

APPENDIX C

BLAST RESISTANT PANEL

PERFORMANCE AND VARIATION OF PERFORMANCE CALCULATIONS

Details regarding the calculation of the deflection, mass, variation of deflection, and variation of mass of blast resistant panels (BRPs) are given below. Information relating to BRPs with three layers (front face sheet, core, back face sheet) are provided first. Then, details regarding BRPs with three solid layers and one solid layer follow. Developing the following BRP performance models is a joint effort with Stephanie Thompson of the Systems Realization Lab at Georgia Tech.

C.1 BLAST RESISTANT PANEL – 3 LAYERS WITH HONEYCOMB CORE

The following section contains equations and explanations for calculating the performance and variation of performance of a BRP with three layers with a honeycomb core. Blast resistant panels discussed in this section contain a solid front face sheet and solid back face sheet with a cellular honeycomb core. The square topology of the core layer is perpendicular to the front and back face sheets. Equations are adapted from the work of Xue and Hutchinson, 2005 in “Metal Sandwich Plates Optimized for Pressure Impulses”. References to the particular page number, equation, or paragraph are presented in given to direct the reader to where the selection equations and / or assumptions are located in the referenced Xue and Hutchinson paper, 2005.

The following equations apply for the following cases only:

- Blasts in air only, not water
- All metal sandwich plate with square honeycomb core.

- The base materials are idealized to be rate-independent and perfectly plastic with yield stresses, $\sigma_{Y,f}$, $\sigma_{Y,c}$, $\sigma_{Y,b}$, for the front face sheet, core, and back face sheet materials, respectively.
- The plate has width 2L, is fully clamped at both ends, and is imagined to be of infinite extent in the y-direction (Xue and Hutchinson 2005 [p 554, last paragraph]).
- The pulse … is taken to be uniform such that at the beginning of Stage III, KE_{II} is uniformly distributed over the plate (Xue and Hutchinson 2005 [p 554, last paragraph]).

The following equations are based on the work of J. W. Hutchinson and Z. Xue in "Metal sandwich plates optimized for pressure impulses" (Xue and Hutchonson 2005). References are made to the page number and equation number of each equation used from the Xue and Hutchinson paper. The nomenclature used here is identical to the nomenclature used in the referenced paper on page 546 in the referenced paper, except for the following:

h_f = front face sheet height
h_b = back face sheet height
$\sigma_{Y,f}$ = front face sheet yield strength
$\sigma_{Y,c}$ = core material yield strength
$\sigma_{Y,b}$ = back face sheet yield strength
ρ_f = front face sheet density
ρ_c = core material density
ρ_b = back face sheet density

C.1.1 Three Stage Analysis of Dynamic Plate Response

Stage I: Fluid-Structure Interaction

For impulses in air: (Xue and Hutchinson 2005 [p 549])

$I_T = 2I_0$

$f_T = 2$

$f_B = 0$

$f_F = 2$

$r_w = 0$

The pressure impulse is characterized as follows:

$$p = p_0 e^{-t/t_0} \tag{C.1}$$

(Xue and Hutchinson 2005 [p 548, paragraph 2])

Therefore, the momentum/area of the free-field pulse is:

$$I_0 = \int p\mathrm{d}t = p_0 t_0 \tag{C.2}$$

(Xue and Hutchinson 2005 [p 548, paragraph 2])

The total kinetic energy/area at the end of Stage I is:

$$KE_I = \frac{I_F^2}{2(m_f + m_w)} + \frac{I_B^2}{2(m_f + m_c)} \tag{C.3}$$

(Xue and Hutchinson 2005 [p 550, equation 5])

in air, $m_w = 0$, $I_B = f_B I_0 = 0$

$$KE_I = \frac{I_F^2}{2(m_f)} = \frac{(2p_0t_0)^2}{2(m_f)} = \frac{4I_0^2}{2(m_f)} \tag{C.4}$$

For independent face sheet thicknesses and material properties:

$$KE_I = \frac{2I_0^2}{(m_f)} \tag{C.5}$$

Stage II: Core Crushing

The total kinetic energy/area at the end of Stage II is:

$$KE_{II} = \frac{I_T^2}{2(2m_f + m_c + m_w)} \tag{C.6}$$

(Xue and Hutchinson 2005 [p 551, equation 6])

For independent face sheet thicknesses and material properties:

$$KE_{II} = \frac{2I_0^2}{(m_f + m_c + m_b)} = \frac{2I_0^2}{(\rho_f h_f + \rho_c R_c H + \rho_b h_b)} \tag{C.7}$$

The kinetic energy/area dissipated by core crushing is:

$$KE_I - KE_{II} = \frac{2I_0^2(m_b + m_c)}{m_f(m_f + m_b + m_c)} \tag{C.8}$$

Obtain the crushing strain, $\bar{\varepsilon}_c$, by equating the plastic dissipation in the core, W_c^P, to the kinetic energy loss in Stage II, KE_I-KE_{II}.

$$W_c^P = \sigma_Y^c \bar{\varepsilon}_c H \tag{C.9}$$

(Xue and Hutchinson 2005 [in text above equation 11 on pg 552])

$$\sigma_Y^c = \lambda_c R_c \sigma_{Y,c} \tag{C.10}$$

(Xue and Hutchinson 2005 [pg 552, equation 10])

$$\sigma_Y^c \bar{\varepsilon}_c H = KE_I - KE_{II} = \frac{2I_0^2 (m_b + m_c)}{m_f (m_f + m_b + m_c)} \tag{C.11}$$

$$\bar{\varepsilon}_c = \frac{2I_0^2 (m_b + m_c)}{\lambda_c R_c \sigma_{Y,c} H m_f (m_f + m_b + m_c)} \tag{C.12}$$

$$\overline{H} = H(1 - \bar{\varepsilon}_c) \tag{C.13}$$

(Xue and Hutchinson 2005 [in text on p 556 after equation 17])

Stage III: Overall Bending and Stretching

"The kinetic energy/area of the plate at the end of Stage II, KEII in Eq. (6), must be dissipated by bending and stretching of the plate in Stage III" (Xue and Hutchinson 2005 [p 554, section 5]).

Plastic work/area dissipated in Stage III is:

$$W_{III}^P = \frac{2}{3}\sigma_Y h_f (2 + \lambda_S \mu)\left(\frac{\delta}{L}\right)^2 + 4\sigma_Y h_f \frac{\overline{H}}{L}\frac{\delta}{L} \tag{C.14}$$

(Xue and Hutchinson 2005 [eq.17, p.556])

$$W_{III}^{P}=\frac{2}{3}\left[\sigma_{Y,f}h_f+\sigma_{Y,c}R_c H\lambda_S+\sigma_{Y,b}h_b\right]\left(\frac{\delta}{L}\right)^2+4\sigma_{Y,b}h_b\frac{\overline{H}}{L}\left(\frac{\delta}{L}\right) \quad \text{(C.15)}$$

C.1.2 Calculating the deflection of the plate

Solve for deflection by equating KE_{II} to W^{P}_{III}, plastic work/area dissipated in Stage III.

$$\begin{aligned} W_{III}^{P}&=\frac{2}{3}\left[\sigma_{Y,f}h_f+\sigma_{Y,c}R_c H\lambda_S+\sigma_{Y,b}h_b\right]\left(\frac{\delta}{L}\right)^2+4\sigma_{Y,b}h_b\frac{\overline{H}}{L}\left(\frac{\delta}{L}\right)= \\ &=KE_{II}=\frac{2I_0^2}{\left(\rho_f h_f+\rho_c R_c H+\rho_b h_b\right)} \end{aligned} \quad \text{(C.16)}$$

Deflection Equation

$$A\left(\frac{\delta}{L}\right)^2+B\left(\frac{\delta}{L}\right)-C=0 \quad \text{(C.17)}$$

$$A=\frac{2}{3}\left[\sigma_{Y,f}h_f+\sigma_{Y,c}R_c H\lambda_S+\sigma_{Y,b}h_b\right] \quad \text{(C.18)}$$

$$B=4\sigma_{Y,b}h_b\frac{\overline{H}}{L} \quad \text{(C.19)}$$

$$C=\frac{-2I_0^2}{\left(\rho_f h_f+\rho_c R_c H+\rho_b h_b\right)} \quad \text{(C.20)}$$

The quadratic equation above is then solved to find the normalized deflection (δ/L). The maximum of the two roots is taken to ensure a positive deflection.

$$\frac{\delta}{L} = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A} \tag{C.21}$$

$$\delta = \frac{-L\left(4\sigma_{y,b}h_b\frac{\overline{H}}{L}\right) \pm L\sqrt{\left(4\sigma_{y,b}h_b\frac{\overline{H}}{L}\right)^2 - 4\left(\frac{2}{3}\left[\sigma_{y,f}h_f + \sigma_{y,c}R_cH\lambda_S + \sigma_{y,b}h_b\right]\right)\left(\frac{-2I_0^2}{\left(\rho_f h_f + \rho_c R_c H + \rho_b h_b\right)}\right)}}{2\left(\frac{2}{3}\left[\sigma_{y,f}h_f + \sigma_{y,c}R_cH\lambda_S + \sigma_{y,b}h_b\right]\right)}$$

(C.22)

$$\delta = \frac{-3\left(4\sigma_{y,b}h_b\overline{H}\right) \pm 3\sqrt{16\sigma_{y,b}^2h_b^2\overline{H}^2 + 16\left(\frac{I_0^2L^2\left[\sigma_{y,f}h_f + \sigma_{y,c}R_cH\lambda_S + \sigma_{y,b}h_b\right]}{3\left(\rho_f h_f + \rho_c R_c H + \rho_b h_b\right)}\right)}}{4\left[\sigma_{y,f}h_f + \sigma_{y,c}R_cH\lambda_S + \sigma_{y,b}h_b\right]} \tag{C.23}$$

$$\delta = \frac{-3\left(\sigma_{y,b}h_b\overline{H}\right) \pm \sqrt{9\sigma_{y,b}^2h_b^2\overline{H}^2 + \left(\frac{3I_0^2L^2\left[\sigma_{y,f}h_f + \sigma_{y,c}R_cH\lambda_S + \sigma_{y,b}h_b\right]}{\left(\rho_f h_f + \rho_c R_c H + \rho_b h_b\right)}\right)}}{\left[\sigma_{y,f}h_f + \sigma_{y,c}R_cH\lambda_S + \sigma_{y,b}h_b\right]}$$

(C.24)

$$\bar{\varepsilon}_c = \frac{2I_0^2\left(m_b + m_c\right)}{\lambda_c R_c \sigma_{Y,c} H m_f \left(m_f + m_b + m_c\right)} \tag{C.25}$$

$$\delta = \frac{-3\left(\sigma_{y,b} h_b \overline{H}\right) \pm \sqrt{9\sigma_{y,b}^2 h_b^2 \overline{H}^2 + \left(\frac{3I_0^2 L^2 \left[\sigma_{y,f} h_f + \sigma_{y,c} R_c H \lambda_S + \sigma_{y,b} h_b\right]}{\left(\rho_f h_f + \rho_c R_c H + \rho_b h_b\right)}\right)}}{\left[\sigma_{y,f} h_f + \sigma_{y,c} R_c H \lambda_S + \sigma_{y,b} h_b\right]} \tag{C.26}$$

where $\overline{H} = H\left(1-\overline{\varepsilon}_c\right) = \left(H - \frac{2I_0^2\left(\rho_b h_b + \rho_c R_c H\right)}{\lambda_c R_c \sigma_{y,c} \rho_f h_f \left(\rho_f h_f + \rho_b h_b + \rho_c R_c H\right)}\right)$

$$\overline{H} = H\left(1-\overline{\varepsilon}_c\right) = \left(H - \frac{2I_0^2\left(\rho_b h_b + \rho_c R_c H\right)}{\lambda_c R_c \sigma_{Y,c} \rho_f h_f \left(\rho_f h_f + \rho_b h_b + \rho_c R_c H\right)}\right) \tag{C.27}$$

$$\overline{H} = H\left(1-\overline{\varepsilon}_c\right) = \left(\frac{H \lambda_c R_c \sigma_{Y,c} \rho_f h_f \left(\rho_f h_f + \rho_b h_b + \rho_c R_c H\right) - 2I_0^2\left(\rho_b h_b + \rho_c R_c H\right)}{\lambda_c R_c \sigma_{Y,c} \rho_f h_f \left(\rho_f h_f + \rho_b h_b + \rho_c R_c H\right)}\right) \tag{C.28}$$

$$\overline{H}^2 = H^2 - \frac{4HI_0^2\left(\rho_b h_b + \rho_c R_c H\right)}{\lambda_c R_c \sigma_{Y,c} \rho_f h_f \left(\rho_f h_f + \rho_b h_b + \rho_c R_c H\right)} + \frac{4I_0^4\left(\rho_b h_b + \rho_c R_c H\right)^2}{\lambda_c^2 R_c^2 \sigma_{Y,c}^2 \rho_f^2 h_f^2 \left(\rho_f h_f + \rho_b h_b + \rho_c R_c H\right)^2} \tag{C.29}$$

C.1.3 Variation of Deflection Calculations

Dividing the Deflection Equation

The deflection equation is split into two function *f*1 and *f*2:

$$\begin{aligned}
&\delta = f_1\left(\sigma_{Y,b},\sigma_{Y,c},\sigma_{Y,f},\rho_b,\rho_c,\rho_f,p_0,t_0\right) \pm f_2\left(\sigma_{Y,b},\sigma_{Y,c},\sigma_{Y,f},\rho_b,\rho_c,\rho_f,p_0,t_0\right)\\
&f_1\left(\sigma_{Y,b},\sigma_{Y,c},\sigma_{Y,f},\rho_b,\rho_c,\rho_f,p_0,t_0\right) = \frac{-3\left(\sigma_{Y,b} h_b\right)\left[H \lambda_c R_c \sigma_{Y,c} \rho_f h_f \left(\rho_f h_f + \rho_b h_b + \rho_c R_c H\right) - 2I_0^2\left(\rho_b h_b + \rho_c R_c H\right)\right]}{\lambda_c R_c \sigma_{Y,c} \rho_f h_f \left(\rho_f h_f + \rho_b h_b + \rho_c R_c H\right)\left(\sigma_{Y,f} h_f + \sigma_{Y,c} R_c H \lambda_S + \sigma_{Y,b} h_b\right)}\\
&f_2\left(\sigma_{Y,b},\sigma_{Y,c},\sigma_{Y,f},\rho_b,\rho_c,\rho_f,p_0,t_0\right) = \frac{\sqrt{9\sigma_{Y,b}^2 h_b^2 \overline{H}^2 + \left(\frac{3I_0^2 L^2\left[\sigma_{Y,f} h_f + \sigma_{Y,c} R_c H \lambda_S + \sigma_{Y,b} h_b\right]}{\left(\rho_f h_f + \rho_c R_c H + \rho_b h_b\right)}\right)}}{\left(\left[\sigma_{Y,f} h_f + \sigma_{Y,c} R_c H \lambda_S + \sigma_{Y,b} h_b\right]\right)}
\end{aligned} \tag{C.30}$$

f_1 is further divided into g_{1N} and g_{1D}

$$f_1\left(\sigma_{Y,b},\sigma_{Y,c},\sigma_{Y,f},\rho_b,\rho_c,\rho_f,p_0,t_0\right)=\frac{g_{1N}\left(\sigma_{Y,b},\sigma_{Y,c},\rho_b,\rho_c,\rho_f,p_0,t_0\right)}{g_{1D}\left(\sigma_{Y,b},\sigma_{Y,c},\sigma_{Y,f},\rho_b,\rho_c,\rho_f\right)}$$
$$g_{1N}\left(\sigma_{Y,b},\sigma_{Y,c},\rho_b,\rho_c,\rho_f,p_0,t_0\right)=\left[-3H\lambda_c R_c h_f h_b \sigma_{Y,c}\sigma_{Y,b}\left(\rho_f^2 h_f+\rho_f\rho_b h_b+\rho_f\rho_c R_c H\right)+6p_0^2 t_0^2\sigma_{Y,b}h_b\left(\rho_b h_b+\rho_c R_c H\right)\right]$$
$$g_{1D}\left(\sigma_{Y,b},\sigma_{Y,c},\sigma_{Y,f},\rho_b,\rho_c,\rho_f\right)=\lambda_c R_c h_f\left(\rho_f^2 h_f+\rho_f\rho_b h_b+\rho_f\rho_c R_c H\right)\left(\sigma_{Y,c}\sigma_{Y,f}h_f+\sigma_{Y,c}^2 R_c H\lambda_S+\sigma_{Y,c}\sigma_{Y,b}h_b\right)$$

(C.31)

f_2 is also divided into g_{2N} and g_{2D}

$$f_2\left(\sigma_{Y,b},\sigma_{Y,c},\sigma_{Y,f},\rho_b,\rho_c,\rho_f,p_0,t_0\right)=\frac{g_{2N}\left(\sigma_{Y,b},\sigma_{Y,c},\sigma_{Y,f},\rho_b,\rho_c,\rho_f,p_0,t_0\right)}{g_{2D}\left(\sigma_{Y,b},\sigma_{Y,c},\sigma_{Y,f}\right)}$$
$$g_{2N}\left(\sigma_{Y,b},\sigma_{Y,c},\sigma_{Y,f},\rho_b,\rho_c,\rho_f,p_0,t_0\right)=\sqrt{9\sigma_{Y,b}^2 h_b^2\bar{H}^2+\left(\frac{3I_0^2L^2\left[\sigma_{Y,f}h_f+\sigma_{Y,c}R_c H\lambda_S+\sigma_{Y,b}h_b\right]}{\left(\rho_f h_f+\rho_c R_c H+\rho_b h_b\right)}\right)}$$
$$g_{2D}=\left(\left[\sigma_{Y,f}h_f+\sigma_{Y,c}R_c H\lambda_S+\sigma_{Y,b}h_b\right]\right)$$

(C.32)

g_{2N} is then further divided into j_{2N} and j_{2D}. j_{2N} is then divided into 6 piece j_{2Na} through j_{2Nf}

$$g_{2N}\left(\sigma_{Y,b},\sigma_{Y,c},\sigma_{Y,f},\rho_b,\rho_c,\rho_f,p_0,t_0\right)=\sqrt{\frac{j_{2N}\left(\sigma_{Y,b},\sigma_{Y,c},\sigma_{Y,f},\rho_b,\rho_c,\rho_f,p_0,t_0\right)}{j_{2D}\left(\sigma_{Y,c},\rho_b,\rho_c,\rho_f\right)}}$$
$$j_{2N}\left(\sigma_{Y,b},\sigma_{Y,c},\sigma_{Y,f},\rho_b,\rho_c,\rho_f,p_0,t_0\right)=j_{2Na}+j_{2Nb}+j_{2Nc}+j_{2Nd}+j_{2Ne}+j_{2Nf}$$
$$j_{2Na}\left(\sigma_{Y,b},\sigma_{Y,c},\rho_b,\rho_c,\rho_f\right)=\left(9h_b^2H^2\lambda_c^2R_c^2h_f^2\sigma_{Y,b}^2\sigma_{Y,c}^2\left(\rho_f^4h_f^2+\rho_f^2\rho_b^2h_b^2+\rho_f^2\rho_c^2R_c^2H^2+2\rho_f^3h_f\rho_b h_b+2\rho_f^3h_f\rho_c R_c H+2\rho_f^2\rho_b h_b\rho_c R_c H\right)\right)$$
$$j_{2Nb}\left(\sigma_{Y,b},\sigma_{Y,c},\rho_b,\rho_c,\rho_f,p_0,t_0\right)=\left(-36h_b^2H\lambda_c R_c h_f p_0^2t_0^2\sigma_{Y,b}^2\sigma_{Y,c}\left(\rho_b h_b\rho_f^2h_f+\rho_f\rho_b^2h_b^2+2\rho_f\rho_b h_b\rho_c R_c H+\rho_f^2h_f\rho_c R_c H+\rho_f\rho_c^2R_c^2H^2\right)\right)$$
$$j_{2Nc}\left(\sigma_{Y,b},\rho_b,\rho_c,p_0,t_0\right)=\left(36h_b^2p_0^4t_0^4\sigma_{Y,b}^2\left(\rho_b^2h_b^2+2\rho_b h_b\rho_c R_c H+\rho_c^2R_c^2H^2\right)\right)$$
$$j_{2Nd}\left(\sigma_{Y,b},\sigma_{Y,c},\sigma_{Y,f},\rho_f,p_0,t_0\right)=3L^2\lambda_c^2R_c^2h_f^2p_0^2t_0^2\rho_f^3\left(\sigma_{Y,f}\sigma_{Y,c}^2h_f^2+\sigma_{Y,c}^3R_c H h_f\lambda_S+\sigma_{Y,c}^2\sigma_{Y,b}h_f h_b\right)$$
$$j_{2Ne}\left(\sigma_{Y,b},\sigma_{Y,c},\sigma_{Y,f},\rho_b,\rho_f,p_0,t_0\right)=3L^2\lambda_c^2R_c^2h_f^2p_0^2t_0^2\rho_f^2\rho_b\left(\sigma_{Y,f}\sigma_{Y,c}^2h_f h_b+\sigma_{Y,c}^3R_c H h_b\lambda_S+\sigma_{Y,c}^2\sigma_{Y,b}h_b^2\right)$$
$$j_{2Nf}\left(\sigma_{Y,b},\sigma_{Y,c},\sigma_{Y,f},\rho_c,\rho_f,p_0,t_0\right)=3L^2\lambda_c^2R_c^2h_f^2p_0^2t_0^2\rho_f^2\rho_c\left(\sigma_{Y,f}\sigma_{Y,c}^2R_c H h_f+\sigma_{Y,c}^3R_c^2H^2\lambda_S+\sigma_{Y,c}^2\sigma_{Y,b}R_c H h_b\right)$$
$$j_{2D}\left(\sigma_{Y,c},\rho_b,\rho_c,\rho_f\right)=L^2\lambda_c^2R_c^2h_f^2\sigma_{Y,c}^2\left(\rho_f^4h_f^2+\rho_f^2\rho_b^2h_b^2+\rho_f^2\rho_c^2R_c^2H^2+2\rho_f^3h_f\rho_b h_b+2\rho_f^3h_f\rho_c R_c H+2\rho_f^2\rho_b h_b\rho_c R_c H\right)$$

(C.33)

Calculating Variation of Deflection

General form of Variation of Deflection Equation:

$$\Delta\delta = \left|\frac{\partial\delta}{\partial\sigma_{Y,b}}\right|\Delta\sigma_{Y,b} + \left|\frac{\partial\delta}{\partial\sigma_{Y,c}}\right|\Delta\sigma_{Y,c} + \left|\frac{\partial\delta}{\partial\sigma_{Y,f}}\right|\Delta\sigma_{Y,f} + \left|\frac{\partial\delta}{\partial\rho_b}\right|\Delta\rho_b + \left|\frac{\partial\delta}{\partial\rho_c}\right|\Delta\rho_c + \left|\frac{\partial\delta}{\partial\rho_f}\right|\Delta\rho_f + \left|\frac{\partial\delta}{\partial p_0}\right|\Delta p_0 + \left|\frac{\partial\delta}{\partial t_0}\right|\Delta t_0 \tag{C.34}$$

$$\frac{\partial\delta}{\partial x} = \frac{g_{1D}\left(\frac{\partial}{\partial x}(g_{1N})\right) - g_{1N}\left(\frac{\partial}{\partial x}(g_{1D})\right)}{(g_{1D})^2} \pm \left(\frac{\frac{\sqrt{j_{2D}}}{2\sqrt{j_{2N}}}\frac{\partial}{\partial x}(j_{2N}) - \frac{\sqrt{j_{2N}}}{2\sqrt{j_{2D}}}\frac{\partial}{\partial x}(j_{2D})}{j_{2D}(g_{2D})} - \frac{\sqrt{\frac{j_{2N}}{j_{2D}}}\left(\frac{\partial}{\partial x}(g_{2D})\right)}{(g_{2D})^2} \right) \tag{C.35}$$

Variation in Deflection:

$$\Delta\delta = \left|\frac{\partial\delta}{\partial\sigma_{Y,b}}\right|\Delta\sigma_{Y,b} + \left|\frac{\partial\delta}{\partial\sigma_{Y,c}}\right|\Delta\sigma_{Y,c} + \left|\frac{\partial\delta}{\partial\sigma_{Y,f}}\right|\Delta\sigma_{Y,f} + \left|\frac{\partial\delta}{\partial\rho_b}\right|\Delta\rho_b + \left|\frac{\partial\delta}{\partial\rho_c}\right|\Delta\rho_c + \left|\frac{\partial\delta}{\partial\rho_f}\right|\Delta\rho_f + \left|\frac{\partial\delta}{\partial p_0}\right|\Delta p_0 + \left|\frac{\partial\delta}{\partial t_0}\right|\Delta t_0 \tag{C.36}$$

$$\delta = \frac{g_{1N}\left(\sigma_{Y,b},\sigma_{Y,c},\rho_b,\rho_c,\rho_f,p_0,t_0\right)}{g_{1D}\left(\sigma_{Y,b},\sigma_{Y,c},\sigma_{Y,f},\rho_b,\rho_c,\rho_f\right)} \pm \frac{\sqrt{\frac{j_{2N}\left(\sigma_{Y,b},\sigma_{Y,c},\sigma_{Y,f},\rho_b,\rho_c,\rho_f,p_0,t_0\right)}{j_{2D}\left(\sigma_{Y,c},\rho_b,\rho_c,\rho_f\right)}}}{g_{2D}\left(\sigma_{Y,b},\sigma_{Y,c},\sigma_{Y,f}\right)} \tag{C.37}$$

$$\frac{\partial \delta}{\partial x} = \frac{g_{1D}\left(\frac{\partial (g_{1N})}{\partial x}\right) - g_{1N}\left(\frac{\partial (g_{1D})}{\partial x}\right)}{(g_{1D})^2} \pm \left(\frac{(g_{2D})\frac{\partial}{\partial x}\left(\sqrt{\frac{j_{2N}}{j_{2D}}}\right)}{(g_{2D})^2} - \frac{\sqrt{\frac{j_{2N}}{j_{2D}}}\left(\frac{\partial (g_{2D})}{\partial x}\right)}{(g_{2D})^2} \right)$$

$$\frac{\partial \delta}{\partial x} = \frac{g_{1D}\left(\frac{\partial}{\partial x}(g_{1N})\right) - g_{1N}\left(\frac{\partial}{\partial x}(g_{1D})\right)}{(g_{1D})^2} \pm \left(\frac{\sqrt{j_{2D}}\frac{\partial}{\partial x}\left(\sqrt{j_{2N}}\right) - \sqrt{j_{2N}}\frac{\partial}{\partial x}\left(\sqrt{j_{2D}}\right)}{j_{2D}(g_{2D})} - \frac{\sqrt{\frac{j_{2N}}{j_{2D}}}\left(\frac{\partial}{\partial x}(g_{2D})\right)}{(g_{2D})^2} \right)$$

$$\frac{\partial \delta}{\partial x} = \frac{g_{1D}\left(\frac{\partial}{\partial x}(g_{1N})\right) - g_{1N}\left(\frac{\partial}{\partial x}(g_{1D})\right)}{(g_{1D})^2} \pm \left(\frac{\frac{\sqrt{j_{2D}}}{2\sqrt{j_{2N}}}\frac{\partial}{\partial x}(j_{2N}) - \frac{\sqrt{j_{2N}}}{2\sqrt{j_{2D}}}\frac{\partial}{\partial x}(j_{2D})}{j_{2D}(g_{2D})} - \frac{\sqrt{\frac{j_{2N}}{j_{2D}}}\left(\frac{\partial}{\partial x}(g_{2D})\right)}{(g_{2D})^2} \right)$$

(C.38)

C.2 BLAST RESISTANT PANEL – 3 SOLID LAYERS

The following section contains equations and explanations for calculating the performance and variation of performance of a BRP with three solid layers. Blast resistant panels discussed in this section contain a solid front face sheet, a solid back face sheet, and a solid core layer with effective material properties. Equations are adapted from the work of Xue and Hutchinson, 2005 in "Metal Sandwich Plates Optimized for Pressure Impulses". References to the particular page number, equation, or paragraph direct the reader to where the selection equations and / or assumptions are located in the referenced Xue and Hutchinson paper. Since there is no core crushing a three solid layer BRP the deflection equations are greatly simplified.

The following equations apply for the following cases only:

- Blasts in air only, not water
- All metal sandwich plate with square honeycomb core.

- The base materials are idealized to be rate-independent and perfectly plastic with yield stresses, $\sigma_{Y,f}$, $\sigma_{Y,c}$, $\sigma_{Y,b}$, for the front face sheet, core, and back face sheet materials, respectively.
- The plate has width 2L, is fully clamped at both ends, and is imagined to be of infinite extent in the y-direction (Xue and Hutchinson 2005 [p 554, last paragraph]).
- The pulse … is taken to be uniform such that at the beginning of Stage III, KE_{II} is uniformly distributed over the plate (Xue and Hutchinson 2005 [p 554, last paragraph]).

The following equations are based on the work of J. W. Hutchinson and Z. Xue in "Metal sandwich plates optimized for pressure impulses" (Xue and Hutchonson 2005). References are made to the page number and equation number of each equation used from the Xue and Hutchinson paper. The nomenclature used here is identical to the nomenclature used in the referenced paper on page 546 in the referenced paper, except for the following:

h_f = front face sheet height
h_b = back face sheet height
$\sigma_{Y,f}$ = front face sheet yield strength
$\sigma_{Y,c}$ = core material yield strength
$\sigma_{Y,b}$ = back face sheet yield strength
ρ_f = front face sheet density
ρ_c = core material density
ρ_b = back face sheet density

C.2.1 Three Stage Analysis of Dynamic Plate Response

Stage I: Fluid-Structure Interaction

For impulses in air: (Xue and Hutchinson 2005 [p 549])

$I_T = 2I_0$

$f_T = 2$

$f_B = 0$

$f_F = 2$

$r_w = 0$

The pressure impulse is characterized as follows:

$$p = p_0 e^{-t/t_0} \tag{C.39}$$

(Xue and Hutchinson 2005 [p 548, paragraph 2])

Therefore, the momentum/area of the free-field pulse is:

$$I_0 = \int p\mathrm{d}t = p_0 t_0 \tag{C.40}$$

(Xue and Hutchinson 2005 [p 548, paragraph 2])

The total kinetic energy/area at the end of Stage I is:

$$KE_I = \frac{I_F^2}{2(m_f + m_w)} + \frac{I_B^2}{2(m_f + m_c)} \tag{C.41}$$

(Xue and Hutchinson 2005 [p 550, equation 5])

in air, $m_w = 0$, $I_B = f_B I_0 = 0$

$$KE_I = \frac{I_F^2}{2(m_f)} = \frac{(2p_0t_0)^2}{2(m_f)} = \frac{4I_0^2}{2(m_f)} \tag{C.42}$$

For independent face sheet thicknesses and material properties:

$$KE_I = \frac{2I_0^2}{(m_f)} \tag{C.43}$$

Stage II: Core Crushing

Since the core layer of the BRP is approximated as a solid panel, there is no core crushing. The revised equation for the kinetic energy/area of the plate at the end of stage II is presented below.

The total kinetic energy/area at the end of Stage II is:

$$KE_{II} = \frac{2I_0^2}{\rho_f h_f + \rho_c H + \rho_b h_b} \tag{C.44}$$

(Xue and Hutchinson 2005 [p 551, equation 6])

Stage III: Overall Bending and Stretching

"The kinetic energy/area of the plate at the end of Stage II, KEII in Eq. (6), must be dissipated by bending and stretching of the plate in Stage III" (Xue and Hutchinson 2005 [p 554, section 5]).

Plastic work/area dissipated in Stage III is:

$$W_{III}^p = \frac{2}{3}\left[\sigma_{y,f} h_f + \sigma_{y,c} H + \sigma_{y,b} h_b\right]\left(\frac{\delta}{L}\right)^2 + \sigma_{y,b} h_b \frac{H}{L}\left(\frac{\delta}{L}\right) \tag{C.45}$$

(Xue and Hutchinson 2005 [eq.16, p.555])

C.2.2 Calculating the deflection of the plate

Solve for deflection by equating KE_{II} to W^{P}_{III}, plastic work/area dissipated in Stage III.

$$\frac{2I_0^2}{\rho_f h_f + \rho_c H + \rho_b h_b} = \frac{2}{3}\left[\sigma_{y,f} h_f + \sigma_{y,c} H + \sigma_{y,b} h_b\right]\left(\frac{\delta}{L}\right)^2 + \sigma_{y,b} h_b \frac{H}{L}\left(\frac{\delta}{L}\right) \quad \text{(C.46)}$$

Deflection Equation

$$A = \frac{2}{3}\left[\sigma_{y,f} h_f + \sigma_{y,c} H + \sigma_{y,b} h_b\right] \quad \textit{(C.47)}$$

$$B = \sigma_{y,b} h_b \frac{H}{L} \quad \text{(C.48)}$$

$$C = -\frac{2I_0^2}{\rho_f h_f + \rho_c H + \rho_b h_b} \quad \text{(C.49)}$$

$$A\left(\frac{\delta}{L}\right)^2 + B\left(\frac{\delta}{L}\right) + C = 0 \quad \text{(C.50)}$$

$$\left(\frac{\delta}{L}\right) = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A} \quad \text{(C.51)}$$

$$\delta = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A} L \quad \text{(C.52)}$$

$$\delta = \frac{-L\left(\sigma_{y,b} h_b \frac{H}{L}\right) \pm L\sqrt{\left(\sigma_{y,b} h_b \frac{H}{L}\right)^2 - 4\left(\frac{2}{3}\left[\sigma_{y,f} h_f + \sigma_{y,c} H + \sigma_{y,b} h_b\right]\right)\left(-\frac{2(p_0 t_0)^2}{\rho_f h_f + \rho_c H + \rho_b h_b}\right)}}{2\left(\frac{2}{3}\left[\sigma_{y,f} h_f + \sigma_{y,c} H + \sigma_{y,b} h_b\right]\right)}$$

(C.53)

$$\delta = \frac{-L\left(\sigma_{y,b} h_b \frac{H}{L}\right)}{2\left(\frac{2}{3}\left[\sigma_{y,f} h_f + \sigma_{y,c} H + \sigma_{y,b} h_b\right]\right)} \pm \frac{L\sqrt{\left(\sigma_{y,b} h_b \frac{H}{L}\right)^2 - 4\left(\frac{2}{3}\left[\sigma_{y,f} h_f + \sigma_{y,c} H + \sigma_{y,b} h_b\right]\right)\left(-\frac{2(p_0 t_0)^2}{\rho_f h_f + \rho_c H + \rho_b h_b}\right)}}{2\left(\frac{2}{3}\left[\sigma_{y,f} h_f + \sigma_{y,c} H + \sigma_{y,b} h_b\right]\right)}$$

(C.54)

$$\delta = \frac{-\sigma_{y.b} h_b H}{\frac{4}{3}\left[\sigma_{y,f} h_f + \sigma_{y,c} H + \sigma_{y,b} h_b\right]} \pm \frac{L\sqrt{\left(\frac{\sigma_{y,b}^2 h_b^2 H^2}{L^2}\right) + \frac{16}{3}\frac{\left[\sigma_{y,f} h_f + \sigma_{y,c} H + \sigma_{y,b} h_b\right]\left(p_0^2 t_0^2\right)}{\rho_f h_f + \rho_c H + \rho_b h_b}}}{\frac{4}{3}\left[\sigma_{y,f} h_f + \sigma_{y,c} H + \sigma_{y,b} h_b\right]}$$

(C.55)

C.2.3 Variation of Deflection Calculations

Dividing the Deflection Equation

The deflection equation is split into two function *f*1 and *f*2:

$$\delta = f_1\left(\sigma_{y,b}, \sigma_{y,c}, \sigma_{y,f}\right) \pm f_2\left(\sigma_{y,b}, \sigma_{y,c}, \sigma_{y,f}, \rho_b, \rho_c, \rho_f, p_0, t_0\right) \tag{C.56}$$

$$f_1\left(\sigma_{y,b}, \sigma_{y,c}, \sigma_{y,f}\right) = \frac{-\sigma_{y.b} h_b H}{\frac{4}{3}\left[\sigma_{y,f} h_f + \sigma_{y,c} H + \sigma_{y,b} h_b\right]} \tag{C.57}$$

$$f_2(\sigma_{y,b},\sigma_{y,c},\sigma_{y,f},\rho_b,\rho_c,\rho_f,p_0,t_0)=\frac{L\sqrt{\left(\frac{\sigma_{y,b}^2 h_b^2 H^2}{L^2}\right)+\frac{16}{3}\frac{\left[\sigma_{y,f}h_f+\sigma_{y,c}H+\sigma_{y,b}h_b\right]\left(p_0^2 t_0^2\right)}{\rho_f h_f+\rho_c H+\rho_b h_b}}}{\frac{4}{3}\left[\sigma_{y,f}h_f+\sigma_{y,c}H+\sigma_{y,b}h_b\right]}$$

(C.58)

The two functions in the deflection equation (f_1 and f_2) are further divided based on numerator and denominator functions (g_{1N}, g_{1D}, g_{2N}, g_{2D}):

$$f_1(\sigma_{y,b},\sigma_{y,c},\sigma_{y,f})=\frac{g_{1N}(\sigma_{y,b})}{g_{1D}(\sigma_{y,b},\sigma_{y,c},\sigma_{y,f})} \tag{C.59}$$

$$g_{1N}(\sigma_{y,b})=-\sigma_{y.b}h_b H \tag{C.60}$$

$$g_{1D}(\sigma_{y,b},\sigma_{y,c},\sigma_{y,f})=\frac{4}{3}\left[\sigma_{y,f}h_f+\sigma_{y,c}H+\sigma_{y,b}h_b\right] \tag{C.61}$$

$$f_2(\sigma_{y,b},\sigma_{y,c},\sigma_{y,f},\rho_b,\rho_c,\rho_f,p_0,t_0)=\frac{g_{2N}(\sigma_{y,b},\sigma_{y,c},\sigma_{y,f},\rho_b,\rho_c,\rho_f,p_0,t_0)}{g_{2D}(\sigma_{y,b},\sigma_{y,c},\sigma_{y,f})} \tag{C.62}$$

$$g_{2N}(\sigma_{y,b},\sigma_{y,c},\sigma_{y,f},\rho_b,\rho_c,\rho_f,p_0,t_0)=L\sqrt{\left(\frac{\sigma_{y,b}^2 h_b^2 H^2}{L^2}\right)+\frac{16}{3}\frac{\left[\sigma_{y,f}h_f+\sigma_{y,c}H+\sigma_{y,b}h_b\right]\left(p_0^2 t_0^2\right)}{\rho_f h_f+\rho_c H+\rho_b h_b}}$$

(C.63)

$$g_{2N}(\sigma_{y,b},\sigma_{y,c},\sigma_{y,f},\rho_b,\rho_c,\rho_f,p_0,t_0)=\sqrt{\frac{3\left(\sigma_{y,b}^2 h_b^2 H^2\right)\left(\rho_f h_f+\rho_c H+\rho_b h_b\right)+16L^2\left[\sigma_{y,f}h_f+\sigma_{y,c}H+\sigma_{y,b}h_b\right]\left(p_0^2 t_0^2\right)}{3L^2\left(\rho_f h_f+\rho_c H+\rho_b h_b\right)}}$$

(C.64)

$$g_{2D}\left(\sigma_{y,b},\sigma_{y,c},\sigma_{y,f}\right)=\frac{4}{3}\left[\sigma_{y,f}h_f+\sigma_{y,c}H+\sigma_{y,b}h_b\right] \tag{C.65}$$

g_{2N} is further divided into j_{2N} and j_{2D}

$$g_{2N}\left(\sigma_{y,b},\sigma_{y,c},\sigma_{y,f}\right)=\sqrt{\frac{j_{2N}\left(\sigma_{y,b},\sigma_{y,c},\sigma_{y,f},\rho_{b},\rho_c,\rho_f,p_0,t_0\right)}{j_{2D}\left(\rho_{b},\rho_c,\rho_f\right)}} \tag{C.66}$$

$$j_{2N}\left(\sigma_{y,b},\sigma_{y,c},\sigma_{y,f},\rho_{b},\rho_c,\rho_f,p_0,t_0\right)=3\left(\sigma_{y,b}^2h_b^2H^2\right)\left(\rho_f h_f+\rho_c H+\rho_b h_b\right)+16L^2\left[\sigma_{y,f}h_f+\sigma_{y,c}H+\sigma_{y,b}h_b\right]\left(p_0^2t_0^2\right) \tag{C.67}$$

$$j_{2D}\left(\rho_{b},\rho_c,\rho_f\right)=3L^2\left(\rho_f h_f+\rho_c H+\rho_b h_b\right) \tag{C.68}$$

Calculating Variation of Deflection

General form of Variation of Deflection Equation:

$$\Delta\delta=\left|\frac{\partial\delta}{\partial\sigma_{Y,b}}\right|\Delta\sigma_{Y,b}+\left|\frac{\partial\delta}{\partial\sigma_{Y,c}}\right|\Delta\sigma_{Y,c}+\left|\frac{\partial\delta}{\partial\sigma_{Y,f}}\right|\Delta\sigma_{Y,f}+\left|\frac{\partial\delta}{\partial\rho_b}\right|\Delta\rho_b+\left|\frac{\partial\delta}{\partial\rho_c}\right|\Delta\rho_c+\left|\frac{\partial\delta}{\partial\rho_f}\right|\Delta\rho_f+\left|\frac{\partial\delta}{\partial p_0}\right|\Delta p_0+\left|\frac{\partial\delta}{\partial t_0}\right|\Delta t_0 \tag{C.69}$$

$$\frac{\partial\delta}{\partial x}=\frac{g_{1D}\left(\frac{\partial}{\partial x}\left(g_{1N}\right)\right)-g_{1N}\left(\frac{\partial}{\partial x}\left(g_{1D}\right)\right)}{\left(g_{1D}\right)^2}\pm\left(\frac{\frac{\sqrt{j_{2D}}}{2\sqrt{j_{2N}}}\frac{\partial}{\partial x}\left(j_{2N}\right)-\frac{\sqrt{j_{2N}}}{2\sqrt{j_{2D}}}\frac{\partial}{\partial x}\left(j_{2D}\right)}{j_{2D}\left(g_{2D}\right)}-\frac{\sqrt{\frac{j_{2N}}{j_{2D}}}\left(\frac{\partial}{\partial x}\left(g_{2D}\right)\right)}{\left(g_{2D}\right)^2}\right) \tag{C.70}$$

Variation in Deflection:

$$\Delta\delta = \left|\frac{\partial\delta}{\partial\sigma_{Y,b}}\right|\Delta\sigma_{Y,b} + \left|\frac{\partial\delta}{\partial\sigma_{Y,c}}\right|\Delta\sigma_{Y,c} + \left|\frac{\partial\delta}{\partial\sigma_{Y,f}}\right|\Delta\sigma_{Y,f} + \left|\frac{\partial\delta}{\partial\rho_b}\right|\Delta\rho_b + \left|\frac{\partial\delta}{\partial\rho_c}\right|\Delta\rho_c + \left|\frac{\partial\delta}{\partial\rho_f}\right|\Delta\rho_f + \left|\frac{\partial\delta}{\partial p_0}\right|\Delta p_0 + \left|\frac{\partial\delta}{\partial t_0}\right|\Delta t_0 \tag{C.71}$$

$$\delta = \frac{g_{1N}\left(\sigma_{Y,b},\sigma_{Y,c},\rho_b,\rho_c,\rho_f,p_0,t_0\right)}{g_{1D}\left(\sigma_{Y,b},\sigma_{Y,c},\sigma_{Y,f},\rho_b,\rho_c,\rho_f\right)} \pm \frac{\sqrt{\dfrac{j_{2N}\left(\sigma_{Y,b},\sigma_{Y,c},\sigma_{Y,f},\rho_b,\rho_c,\rho_f,p_0,t_0\right)}{j_{2D}\left(\sigma_{Y,c},\rho_b,\rho_c,\rho_f\right)}}}{g_{2D}\left(\sigma_{Y,b},\sigma_{Y,c},\sigma_{Y,f}\right)} \tag{C.72}$$

$$\frac{\partial\delta}{\partial x} = \frac{g_{1D}\left(\dfrac{\partial(g_{1N})}{\partial x}\right) - g_{1N}\left(\dfrac{\partial(g_{1D})}{\partial x}\right)}{\left(g_{1D}\right)^2} \pm \left(\frac{\left(g_{2D}\right)\dfrac{\partial}{\partial x}\left(\sqrt{\dfrac{j_{2N}}{j_{2D}}}\right)}{\left(g_{2D}\right)^2} - \frac{\sqrt{\dfrac{j_{2N}}{j_{2D}}}\left(\dfrac{\partial(g_{2D})}{\partial x}\right)}{\left(g_{2D}\right)^2}\right)$$

$$\frac{\partial\delta}{\partial x} = \frac{g_{1D}\left(\dfrac{\partial}{\partial x}(g_{1N})\right) - g_{1N}\left(\dfrac{\partial}{\partial x}(g_{1D})\right)}{\left(g_{1D}\right)^2} \pm \left(\frac{\sqrt{j_{2D}}\dfrac{\partial}{\partial x}\left(\sqrt{j_{2N}}\right) - \sqrt{j_{2N}}\dfrac{\partial}{\partial x}\left(\sqrt{j_{2D}}\right)}{j_{2D}\left(g_{2D}\right)} - \frac{\sqrt{\dfrac{j_{2N}}{j_{2D}}}\left(\dfrac{\partial}{\partial x}(g_{2D})\right)}{\left(g_{2D}\right)^2}\right)$$

$$\frac{\partial\delta}{\partial x} = \frac{g_{1D}\left(\dfrac{\partial}{\partial x}(g_{1N})\right) - g_{1N}\left(\dfrac{\partial}{\partial x}(g_{1D})\right)}{\left(g_{1D}\right)^2} \pm \left(\frac{\dfrac{\sqrt{j_{2D}}}{2\sqrt{j_{2N}}}\dfrac{\partial}{\partial x}\left(j_{2N}\right) - \dfrac{\sqrt{j_{2N}}}{2\sqrt{j_{2D}}}\dfrac{\partial}{\partial x}\left(j_{2D}\right)}{j_{2D}\left(g_{2D}\right)} - \frac{\sqrt{\dfrac{j_{2N}}{j_{2D}}}\left(\dfrac{\partial}{\partial x}(g_{2D})\right)}{\left(g_{2D}\right)^2}\right) \tag{C.73}$$

C.3 BLAST RESISTANT PANEL – 1 SOLID LAYER

The following section contains equations and explanations for calculating the performance and variation of performance of a BRP with one solid layer. Blast resistant panels discussed in this section contain one solid with effective material properties. Equations are adapted from the work of Xue and Hutchinson, 2005 in “Metal Sandwich Plates Optimized for Pressure Impulses”. References to the particular page number, equation, or paragraph direct the reader to where the selection equations and / or assumptions are located in the referenced Xue and Hutchinson paper.

The following equations apply for the following cases only:

- Blasts in air only, not water
- All metal sandwich plate with square honeycomb core.
- The base materials are idealized to be rate-independent and perfectly plastic
- The plate has width 2L, is fully clamped at both ends, and is imagined to be of infinite extent in the y-direction (Xue and Hutchinson 2005 [p 554, last paragraph]).
- The pulse … is taken to be uniform such that at the beginning of Stage III, KE_{II} is uniformly distributed over the plate (Xue and Hutchinson 2005 [p 554, last paragraph]).

The following equations are based on the work of J. W. Hutchinson and Z. Xue in "Metal sandwich plates optimized for pressure impulses" (Xue and Hutchonson 2005). References are made to the page number and equation number of each equation used from the Xue and Hutchinson paper. The nomenclature used here is identical to the nomenclature used in the referenced paper on page 546 in the referenced paper, except for the following:

h = height of the panel
σ_Y = yield strength
ρ = density

C.3.1 Three Stage Analysis of Dynamic Plate Response

Stage I: Fluid-Structure Interaction

For impulses in air: (Xue and Hutchinson 2005 [p 549])

$I_T = 2I_0$

$f_T = 2$

$f_B = 0$

$f_F = 2$

$r_w = 0$

The pressure impulse is characterized as follows:

$$p = p_0 e^{-t/t_0} \tag{C.74}$$

(Xue and Hutchinson 2005 [p 548, paragraph 2])

Therefore, the momentum/area of the free-field pulse is:

$$I_0 = \int p\mathrm{d}t = p_0 t_0 \tag{C.75}$$

(Xue and Hutchinson 2005 [p 548, paragraph 2])

Stage II: Core Crushing

Since the core layer of the BRP is approximated as a solid panel, there is no core crushing. The revised equation for the kinetic energy/area of the plate at the end of stage II is presented below.

The total kinetic energy/area at the end of Stage II is:

$$KE_{II} = \frac{2I_0^2}{\rho h} \tag{C.76}$$

(Xue and Hutchinson 2005 [p 551, equation 6])

Stage III: Overall Bending and Stretching

"The kinetic energy/area of the plate at the end of Stage II, KEII in Eq. (6), must be dissipated by bending and stretching of the plate in Stage III" (Xue and Hutchinson 2005 [p 554, section 5]).

Plastic work/area dissipated in Stage III is:

$$W_{III}^{p} = \frac{2}{3}\left[\sigma_y h\left(\frac{\delta}{L}\right)^2 + \sigma_y h\frac{h}{L}\left(\frac{\delta}{L}\right)\right] \tag{C.77}$$

(Xue and Hutchinson 2005 [eq.16, p.555])

C.3.2 Calculating the deflection of the plate

Solve for deflection by equating KE_{II} to W^{P}_{III}, plastic work/area dissipated in Stage III.

$$\frac{2I_0^2}{\rho h} = \frac{2}{3}\left[\sigma_y h\left(\frac{\delta}{L}\right)^2 + \sigma_y h\frac{h}{L}\left(\frac{\delta}{L}\right)\right] \tag{C.78}$$

Deflection Equation

$$A = \frac{2}{3}\left[\sigma_y h\right] \tag{C.79}$$

$$B = \sigma_y \frac{h^2}{L} \tag{C.80}$$

$$C = -\frac{2I_0^2}{\rho h} \tag{C.81}$$

$$A\left(\frac{\delta}{L}\right)^2 + B\left(\frac{\delta}{L}\right) + C = 0 \tag{C.82}$$

$$\left(\frac{\delta}{L}\right) = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A} \tag{C.83}$$

$$\delta = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A} L \tag{C.84}$$

$$\delta = \frac{-\left(\sigma_y \frac{h^2}{L}\right) \pm \sqrt{\left(\sigma_y \frac{h^2}{L}\right)^2 - 4\left(\frac{2}{3}\left[\sigma_y h\right]\right)\left(-\frac{2I_0^2}{\rho h}\right)}}{2\left(\frac{2}{3}\left[\sigma_y h\right]\right)} L \tag{C.85}$$

$$\delta = \frac{-L\left(\sigma_y \frac{h^2}{L}\right)}{2\left(\frac{2}{3}[\sigma_y h]\right)} \pm \frac{L\sqrt{\left(\sigma_y \frac{h^2}{L}\right)^2 - 4\left(\frac{2}{3}[\sigma_y h]\right)\left(-\frac{2I_0^2}{\rho h}\right)}}{2\left(\frac{2}{3}[\sigma_y h]\right)} \tag{C.86}$$

$$\delta = \frac{-\sigma_y h^2}{\frac{4}{3}[\sigma_y h]} \pm \frac{L\sqrt{\left(\frac{\sigma_y^2 h^4}{L^2}\right) + \frac{16}{3}\frac{[\sigma_y h](p_0^2 t_0^2)}{\rho h}}}{\frac{4}{3}[\sigma_y h]} \tag{C.87}$$

C.3.3 Variation of Deflection Calculations

Dividing the Deflection Equation

The deflection equation is divided into two functions f_1 and f_2:

$$\delta = f_1(\sigma_y) \pm f_2(\sigma_y, \rho, p_0, t_0) \tag{C.88}$$

$$f_1 = \frac{-\sigma_y h^2}{\frac{4}{3}[\sigma_y h]} \tag{C.89}$$

$$f_2 = \frac{L\sqrt{\left(\frac{\sigma_y^2 h^4}{L^2}\right) + \frac{16}{3}\frac{[\sigma_y h](p_0^2 t_0^2)}{\rho h}}}{\frac{4}{3}[\sigma_y h]} \tag{C.90}$$

The two functions in the deflection equation (f_1 and f_2) are further divided based on numerator and denominator functions (g_{1N}, g_{1D}, g_{2N}, g_{2D}):

$$f_1(\sigma_y) = \frac{g_{1N}(\sigma_y)}{g_{1D}(\sigma_y)} \tag{C.91}$$

$$g_{1N}(\sigma_y) = -\sigma_y h^2 \tag{C.92}$$

$$g_{1D}(\sigma_y) = \frac{4}{3}[\sigma_y h] \tag{C.93}$$

$$f_2(\sigma_y, \rho, p_0, t_0) = \frac{g_{2N}(\sigma_y, \rho, p_0, t_0)}{g_{2D}(\sigma_y)} \tag{C.94}$$

$$g_{2N}(\sigma_y, \rho, p_0, t_0) = L\sqrt{\left(\frac{\sigma_y^2 h^4}{L^2}\right) + \frac{16}{3}\frac{[\sigma_y h](p_0^2 t_0^2)}{\rho h}} \tag{C.95}$$

$$g_{2N}(\sigma_y, \rho, p_0, t_0) = L\sqrt{\frac{3(\sigma_y^2 h^4)(\rho h) + 16L^2[\sigma_y h](p_0^2 t_0^2)}{3L^2 \rho h}} \tag{C.96}$$

$$g_{2D}(\sigma_y) = \frac{4}{3}[\sigma_y h] \tag{C.97}$$

g_{2N} is further divided into j_{2N} and j_{2D}

$$g_{2N}(\sigma_y) = \sqrt{\frac{j_{2N}(\sigma_y, \rho, p_0, t_0)}{j_{2D}(\rho)}} \tag{C.98}$$

$$j_{2N}(\sigma_y, \rho, p_0, t_0) = 3(\sigma_y^2 h^4)(\rho h) + 16L^2[\sigma_y h](p_0^2 t_0^2) \tag{C.99}$$

$$j_{2D}(\rho) = 3L^2 \rho h \tag{C.100}$$

Calculating Variation of Deflection

General form of Variation of Deflection Equation:

$$\Delta\delta = \left|\frac{\partial\delta}{\partial\sigma_{Y,b}}\right|\Delta\sigma_{Y,b} + \left|\frac{\partial\delta}{\partial\sigma_{Y,c}}\right|\Delta\sigma_{Y,c} + \left|\frac{\partial\delta}{\partial\sigma_{Y,f}}\right|\Delta\sigma_{Y,f} + \left|\frac{\partial\delta}{\partial\rho_b}\right|\Delta\rho_b + \left|\frac{\partial\delta}{\partial\rho_c}\right|\Delta\rho_c + \left|\frac{\partial\delta}{\partial\rho_f}\right|\Delta\rho_f + \left|\frac{\partial\delta}{\partial p_0}\right|\Delta p_0 + \left|\frac{\partial\delta}{\partial t_0}\right|\Delta t_0 \tag{C.101}$$

$$\frac{\partial\delta}{\partial x} = \frac{g_{1D}\left(\frac{\partial}{\partial x}(g_{1N})\right) - g_{1N}\left(\frac{\partial}{\partial x}(g_{1D})\right)}{(g_{1D})^2} \pm \left(\frac{\frac{\sqrt{j_{2D}}}{2\sqrt{j_{2N}}}\frac{\partial}{\partial x}(j_{2N}) - \frac{\sqrt{j_{2N}}}{2\sqrt{j_{2D}}}\frac{\partial}{\partial x}(j_{2D})}{j_{2D}(g_{2D})} - \frac{\sqrt{\frac{j_{2N}}{j_{2D}}}\left(\frac{\partial}{\partial x}(g_{2D})\right)}{(g_{2D})^2} \right) \tag{C.102}$$

Variation in Deflection:

$$\Delta\delta = \left|\frac{\partial\delta}{\partial\sigma_{Y,b}}\right|\Delta\sigma_{Y,b} + \left|\frac{\partial\delta}{\partial\sigma_{Y,c}}\right|\Delta\sigma_{Y,c} + \left|\frac{\partial\delta}{\partial\sigma_{Y,f}}\right|\Delta\sigma_{Y,f} + \left|\frac{\partial\delta}{\partial\rho_b}\right|\Delta\rho_b + \left|\frac{\partial\delta}{\partial\rho_c}\right|\Delta\rho_c + \left|\frac{\partial\delta}{\partial\rho_f}\right|\Delta\rho_f + \left|\frac{\partial\delta}{\partial p_0}\right|\Delta p_0 + \left|\frac{\partial\delta}{\partial t_0}\right|\Delta t_0 \tag{C.103}$$

$$\delta = \frac{g_{1N}\left(\sigma_{Y,b},\sigma_{Y,c},\rho_b,\rho_c,\rho_f,p_0,t_0\right)}{g_{1D}\left(\sigma_{Y,b},\sigma_{Y,c},\sigma_{Y,f},\rho_b,\rho_c,\rho_f\right)} \pm \frac{\sqrt{\frac{j_{2N}\left(\sigma_{Y,b},\sigma_{Y,c},\sigma_{Y,f},\rho_b,\rho_c,\rho_f,p_0,t_0\right)}{j_{2D}\left(\sigma_{Y,c},\rho_b,\rho_c,\rho_f\right)}}}{g_{2D}\left(\sigma_{Y,b},\sigma_{Y,c},\sigma_{Y,f}\right)} \tag{C.104}$$

$$\frac{\partial \delta}{\partial x} = \frac{g_{1D}\left(\frac{\partial\left(g_{1N}\right)}{\partial x}\right) - g_{1N}\left(\frac{\partial\left(g_{1D}\right)}{\partial x}\right)}{\left(g_{1D}\right)^2} \pm \left(\frac{\left(g_{2D}\right)\frac{\partial}{\partial x}\left(\sqrt{\frac{j_{2N}}{j_{2D}}}\right)}{\left(g_{2D}\right)^2} - \frac{\sqrt{\frac{j_{2N}}{j_{2D}}}\left(\frac{\partial\left(g_{2D}\right)}{\partial x}\right)}{\left(g_{2D}\right)^2} \right)$$

$$\frac{\partial \delta}{\partial x} = \frac{g_{1D}\left(\frac{\partial}{\partial x}\left(g_{1N}\right)\right) - g_{1N}\left(\frac{\partial}{\partial x}\left(g_{1D}\right)\right)}{\left(g_{1D}\right)^2} \pm \left(\frac{\sqrt{j_{2D}}\frac{\partial}{\partial x}\left(\sqrt{j_{2N}}\right) - \sqrt{j_{2N}}\frac{\partial}{\partial x}\left(\sqrt{j_{2D}}\right)}{j_{2D}\left(g_{2D}\right)} - \frac{\sqrt{\frac{j_{2N}}{j_{2D}}}\left(\frac{\partial}{\partial x}\left(g_{2D}\right)\right)}{\left(g_{2D}\right)^2} \right)$$

$$\frac{\partial \delta}{\partial x} = \frac{g_{1D}\left(\frac{\partial}{\partial x}\left(g_{1N}\right)\right) - g_{1N}\left(\frac{\partial}{\partial x}\left(g_{1D}\right)\right)}{\left(g_{1D}\right)^2} \pm \left(\frac{\frac{\sqrt{j_{2D}}}{2\sqrt{j_{2N}}}\frac{\partial}{\partial x}\left(j_{2N}\right) - \frac{\sqrt{j_{2N}}}{2\sqrt{j_{2D}}}\frac{\partial}{\partial x}\left(j_{2D}\right)}{j_{2D}\left(g_{2D}\right)} - \frac{\sqrt{\frac{j_{2N}}{j_{2D}}}\left(\frac{\partial}{\partial x}\left(g_{2D}\right)\right)}{\left(g_{2D}\right)^2} \right)$$

(C.105)

APPENDIX D

VERIFICATION AND VALIDATION OF COMPUTATIONAL DESIGN TOOLS IN BRP DESIGN

In the following section, a look at the validity of the computational design tools used in calculating BRP design solutions is presented by addressing the following topics: BRP computational models, BRP finite element analysis, and BRP impulse loading analysis. The BRP computational models were developed in conjunction with Stephanie Thompson of the Systems Realization Lab at Georgia Tech. The BRP finite element analysis and BRP impulse loading analysis was lead by Jin Song, a former undergraduate researcher in the Systems Realization Lab at Georgia Tech. The intellectual contributions of Stephanie and Jin are essential to the development and verification of BRP performance analysis models.

Computational Models

The computational models are BRP performance prediction tools implemented in MATLAB and used to predict the performance of a BRP designed at various levels of model complexity. The equations used in generating BRP computational models are based on the work of Hutchinson and Xue (Hutchinson and Xue 2005). Equations for calculating the deflection and energy absorption of a BRP are provided in detail in Appendix C. The BRP performance computational models are one aspect of a BRP multilevel design template. In addition to the computational models, design goals, constraints, bounds, preferences, mapping functions and a solution finding algorithm are used to form a domain-specific, computer executable design template for the multilevel robust design of BRPs.

Confidence is built in the validity of the computational models due to the fact that they are based on existing performance calculations of BRPs found in the literature (Hutchinson and Xue 2005). Although the equations for BRP deflection are taken directly from the work of Hutchinson and Xue, BRP deflection solutions calculated as part of the research in this thesis do not match the deflection results published by Hutchinson and Xue (Hutchinson and Xue 2005). Deflection calculations based on the work in this thesis disagree with published deflection calculations by approximately an order of 2. After rigorous examination for possible errors, researches in the Systems Realization Lab at Georgia Tech are confident that the BRP performance equations have been implemented correctly. Due to this inconsistency, researchers in the SRL at Georgia Tech have contacted Hutchinson and Xue in an effort to resolve this in consistency. Since this issue is not yet resolved, it is left as future work in the BRP design project.

Finite Element Analysis

To provide further validation to the computational design tools used to predict BRP performance, finite element analysis (FEA) of BRP performance using the commercial software package, ABAQUS (ABAQUS 2005), is currently being conducted. Information regarding BRP analysis using ABAQUS is contributed by Jin Song a former undergraduate researcher in the Systems Realization Lab at Georgia Tech. Current BRP analysis using ABAQUS is conducted by Gautam Puri, undergraduate researcher in the Systems Realization Lab at Georgia Tech. Once a BRP design solution is obtained, a 3D model of the designed BRP is imported to ABAQUS for further deformation analysis. An example of a BRP analyzed in ABAQUS is shown in Figure D.1.

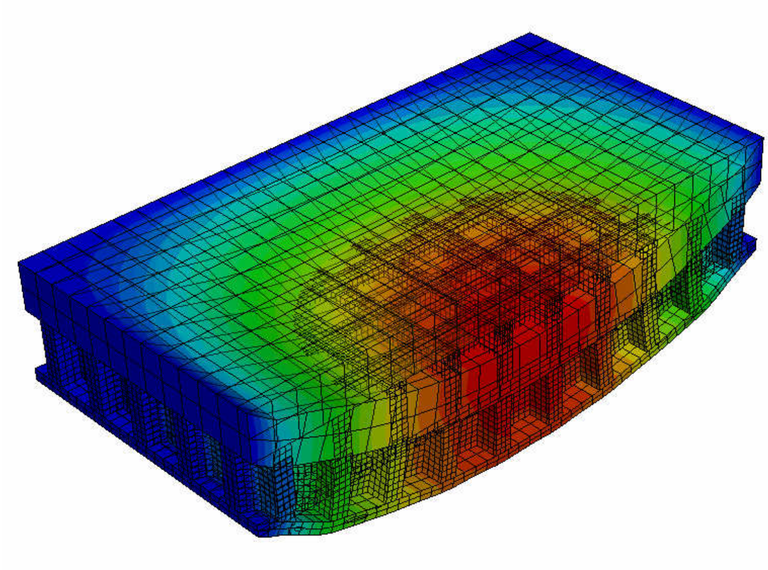


Figure D.1 – BRP deflection analyzed in ABAQUS (contributed by Jin Song)

Current efforts to validate the mathematical models used to predict BRP performance using FEA are in progress. Modeling material behavior in a material modeling FEA program is a significant research challenge requiring many iterations. No FEA solutions are currently available for BRP performance comparison, and this portion of model validation will be addressed in the future work of BRP design.

Analyzing BRP performance using FEA will become extremely valuable in the future when BRP design continues to increase in complexity. For example, there is a research interest in filling various cells in the BRP core layer with a ceramic particle powder. It is assumed that this will further decrease BRP deflection without significantly increasing panel mass. Analyzing complex phenomena such as energy dissipation in ceramic particles will most likely be completed using FEA software.

BRP Impulse Loading Analysis

A close approximation for impulse load is modeled as a spherical wave, shown in the following equation, where θ is the angle of incidence, p_r is the reflected pressure, p_i is the incident pressure, and p_0 is the average peak pressure (Neuberger, et al. 2006).

$$p(t) = p_{\mathrm{r}} \cos^2(\theta) + p_{\mathrm{i}}\left(1 + \cos^2(\theta) - 2\cos(\theta)\right) \tag{D.1}$$

However, the computational models used in BRP deflection calculations implemented in this thesis assume a uniform pressure wave across the front face sheet of the panel, varying as a function of time. The pressure vs. time equation implemented in BRP loading in this thesis is presented below.

$$p(t) = p_0 e^{-t/t_0} \tag{D.2}$$

The uniform pressure wave assumption used in this thesis is compared to a spherical pressure wave in order to provide additional validation of the BRP performance models. Uniform and spherical pressure waves are compared by loading a single solid panel in ABAQUS, and measuring maximum deflection of the panel. An example of a loaded single panel is shown in Figure D.2.

A single solid panel is modeled in ABAQUS and loaded using a uniform pressure wave with peak pressure P_0 and a spherical pressure wave with peak pressure P_0. The maximum deflection of the panel is calculated. The material properties of the panel are modeled as rolled homogeneous armor (RHA) steel in half of the simulations and magnesium alloy in half of the simulations. The purpose of this experiment is to determine the difference in deflection when a panel undergoes uniform versus spherical loading, assuming equal peak pressure in each loading scenario. A secondary goal in this experiment is to determine to role of material properties in a comparison of uniform versus spherical pressure loading.

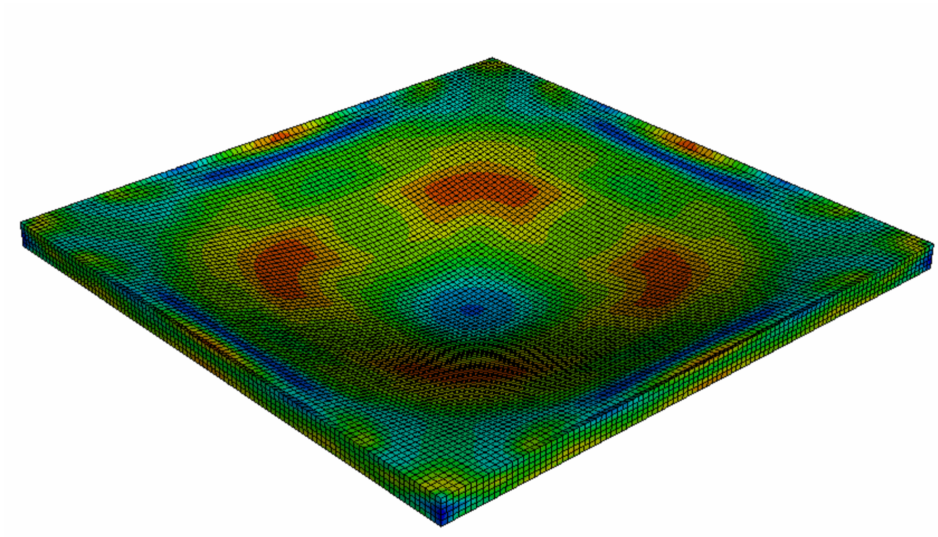


Figure D.2 – Single panel deflection analyzed in ABAQUS (Contributed by Jin Song)

Results from this experiment are shown in Figure D.3. For peak pressure values less than 100 MPa, uniform pressure waves produce greater deflection in both RHA steel and Mg alloy plates. However, at a peak pressure of 200 MPa, it is observed that for a RHA steel panel, a spherical pressure produces greater panel deflection than a uniform pressure wave. This anomaly in the data should be investigated further to see if material properties and peak pressure influence the relationship in deflection of a panel under uniform and spherical loading.

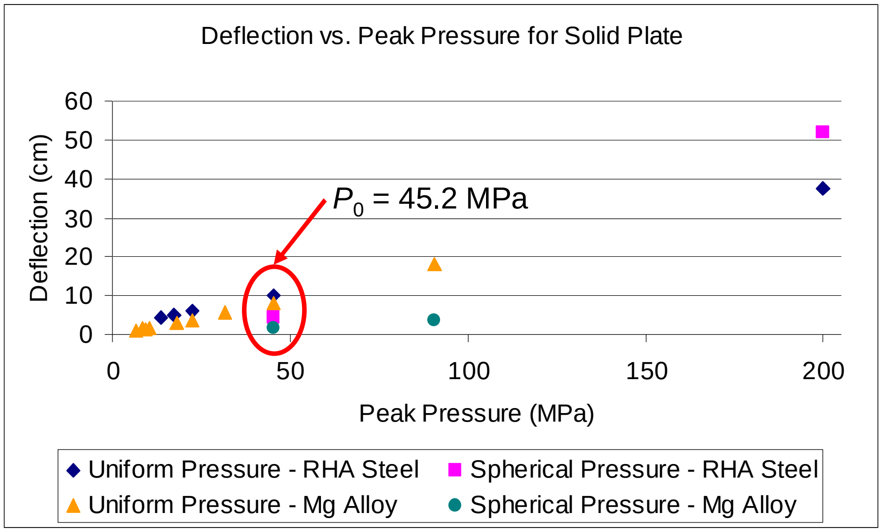


Figure D.3 – A comparison of uniform and spherical loading conditions on a solid plate (Contributed by Jin Song)

After analyzing the results, it is obvious that for large values of peak pressure (approximately greater than 45 MPa), a uniform pressure wave is not a good approximation for a spherical pressure wave. Therefore, further investigation is conducted to more accurately approximate spherical waves using uniform waves by multiplying the peak pressure by a certain scaling factor. The graphs in Figure D.4 show a comparison of spherical and uniform pressure waves on a single solid Mg alloy panel. Notice that a spherical peak pressure of $P_{0,spherical}$ = 90.4 MPa and a uniform peak pressure of $P_{0,uniform}$ = 22.6 MPa result in similar values for maximum panel deflection. This indicates that it is possible that a scaling factor can be applied to uniform pressure calculations in order to more accurately model a spherical pressure load.

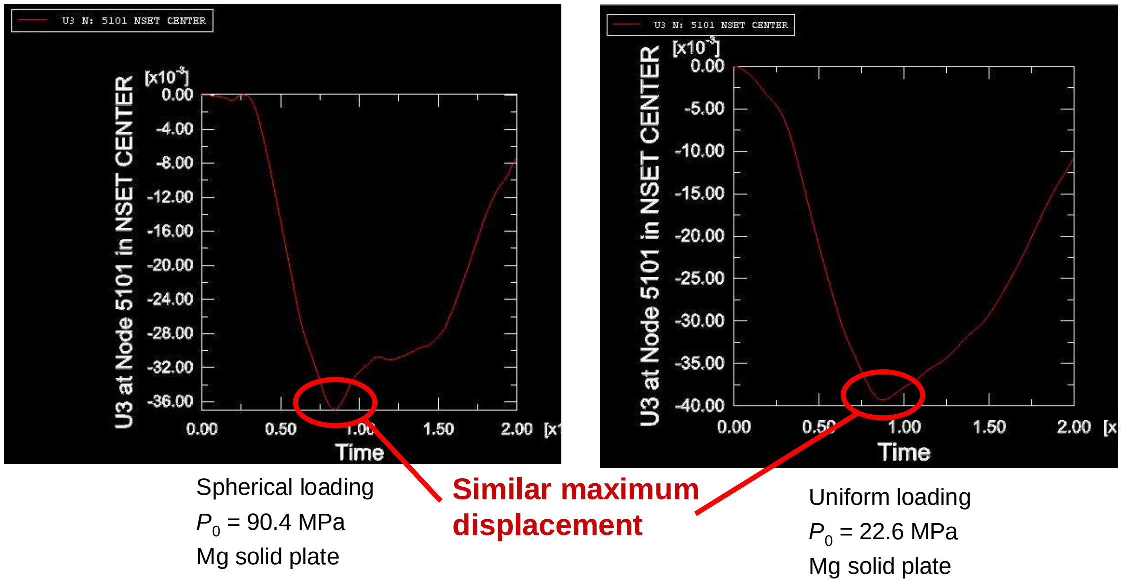


Figure D.4 – A comparison of peak pressure values for uniform and spherical loading conditions resulting in similar panel deflection (Contributed by Jin Song)

APPENDIX E

DATA POINTS FOR BRP PARETO CURVES

In Appendix E, data points used to describe BRP Pareto curves in Figure 5.17 are presented. Outliers for each data set are highlighted in grey.

Table E.1 – Pareto curve data points supporting Figure 5.17

GOAL WEIGHTING		PERFORMANCE		
Deflection	**Mass**	**Deflection (cm)**	**Mass (kg/m^2)**	Figure 5.17 (a)
1	0	1.4476	130.9667	
0.9	0.1	1.4364	130.9699	
0.8	0.2	1.4441	130.9665	
0.7	0.3	1.4493	130.9658	
0.6	0.4	1.4500	130.9636	
0.5	0.5	1.4503	130.9635	
0.4	0.6	1.4491	130.9637	
0.3	0.7	1.4506	130.9635	
0.2	0.8	1.4496	130.9741	
0.1	0.9	1.4448	130.8562	
0	1	1.463	130.5252	
Deflection	**HD-EMI$_\delta$**	**Deflection (cm)**	**Variation of deflection (cm)**	Figure 5.17 (b)
1	0	1.4476	0.9132	
0.9	0.1	1.4504	0.9146	
0.8	0.2	1.4505	0.9147	
0.7	0.3	1.4505	0.9147	
0.6	0.4	1.4500	0.9144	
0.5	0.5	1.4482	0.9135	
0.4	0.6	1.4474	0.9130	
0.3	0.7	1.4498	0.9143	
0.2	0.8	1.446	0.9028	
0.1	0.9	1.4361	0.9017	
0	1	1.433	0.9051	
Mass	**HD-EMI$_M$**	**Mass (kg/m^2)**	**Variation of mass (kg/m^2)**	Figure 5.17 (c)
1	0	130.5252	18.9747	
0.9	0.1	130.5665	18.9761	
0.8	0.2	130.2430	18.9284	
0.7	0.3	130.5773	18.9792	
0.6	0.4	130.5013	18.9656	
0.5	0.5	130.7811	19.0094	

Table E.1 (continued) – Pareto curve data points supporting Figure 5.17

0.4	0.6	125.0214	16.8977	
0.3	0.7	130.8203	19.0139	
0.2	0.8	130.8432	19.0176	
0.1	0.9	130.8748	19.0148	
0	1	130.9941	19.0056	
Deflection	**HD-EMI$_M$**	**Deflection (cm)**	**Variation of mass (kg/m^2)**	Figure 5.17 (d)
1	0	1.4476	19.0333	
0.9	0.1	1.4494	19.0343	
0.8	0.2	1.4475	19.0329	
0.7	0.3	1.4493	19.0340	
0.6	0.4	1.4500	19.0361	
0.5	0.5	1.4503	19.0361	
0.4	0.6	1.4505	19.0361	
0.3	0.7	1.4495	19.0359	
0.2	0.8	1.4512	19.0361	
0.1	0.9	1.4515	19.0362	
0	1	1.455	19.0056	
Mass	**HD-EMI$_\delta$**	**Mass (kg/m^2)**	**Variation of deflection (cm)**	Figure 5.17 (e)
1	0	130.5252	0.9215	
0.9	0.1	130.9749	0.9156	
0.8	0.2	130.9643	0.9152	
0.7	0.3	130.9650	0.9152	
0.6	0.4	130.9658	0.9151	
0.5	0.5	130.9664	0.9152	
0.4	0.6	130.9679	0.9151	
0.3	0.7	131.1077	0.9030	
0.2	0.8	131.0096	0.9120	
0.1	0.9	131.0159	0.8991	
0	1	130.9711	0.9051	
HD-EMI$_\delta$	**HD-EMI$_M$**	**Variation of deflection (cm)**	**Variation of mass (kg/m^2)**	Figure 5.17 (f)
1	0	0.9051	19.0289	
0.9	0.1	0.9115	19.1190	
0.8	0.2	0.9120	18.9716	
0.7	0.3	0.9118	19.1228	
0.6	0.4	0.8597	18.9796	
0.5	0.5	0.9152	19.0332	
0.4	0.6	0.9152	19.0336	
0.3	0.7	0.9152	19.0343	
0.2	0.8	0.9153	19.0345	
0.1	0.9	0.9154	19.0356	
0	1	0.9167	19.0056	

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